EE205 Lecture Notes:

P1

26 Eylül 2020 Cumartesi 17:16

Electric potential (voltage): Work done by charges $:= \frac{E}{q}$

E = Electric potential energy. (Joules) = Work capacity $\downarrow$ $F \cdot d$.

q = charge. (Coulombs)

Current $= \frac{q}{t}$ (A) $= I$

q = charge.

t = time. (sec.)

Power $= \frac{E}{t}$ (watts). $= \underbrace{\frac{E}{q}}_{V} \cdot \underbrace{\frac{q}{t}}_{I} = V \cdot I$ (W).

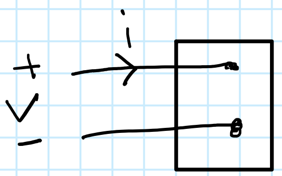
Ideal Basic Circuit Elements:

It is an electrical component with the following properties:

1-) Two terminals.

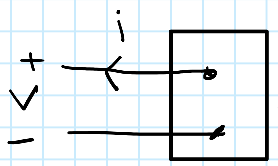
2-) Can be described as voltage or current.

3-) Can not be subdivided into other elements.



$P = V \cdot i$

Symbol for ideal circuit element.



$\boxed{P<0}$

Energy source device.

If the current is going out of the circuit element, this refers to the existence of an energy source (generator).

If the power consumption by the element is positive, ($P>0$), the power is being delivered to the circuit inside the box.

If the power is negative, $P<0$, the power is being extracted from the circuit inside the box.

P2

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Chapter 2 : Circuit Elements :

There are two circuit elements, voltage and current sources.

— Ideal voltage source: Provides a constant voltage across its terminals regardless of the current.

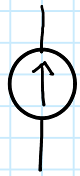
— Ideal current source: Similar to the voltage source. It provides current across its terminals regardless of the voltage.

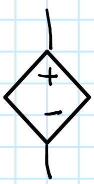
If circuit elements do not depend on any other parameter, they are called "independent sources".

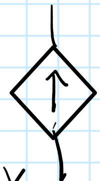
Circuit symbols:

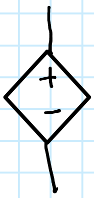
V_s

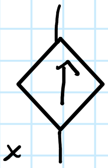
Ideal Independent voltage source.

i_s

Ideal indep. current source.

$v_s = \mu \cdot V_x$

Ideal dependent voltage controlled voltage source.

i_s

$i_s = \alpha \cdot V_x$
Ideal dependent voltage controlled current source.

v_s

$v_s = \rho \cdot i_x$
Ideal dependent current controlled voltage source.


$i_s = \beta \cdot i_x$
Ideal dependent current controlled current source.

P2a

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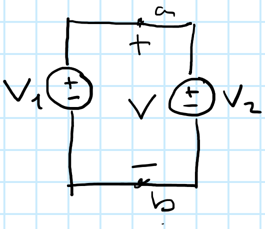
Constraints for the Connection of Circuit Elements:

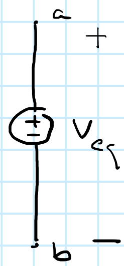
– Two voltage sources:



$\Rightarrow \boxed{V_1 = V_2} = V$

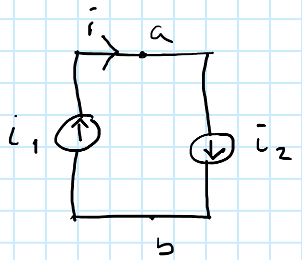
Also, the polarities must be the same.

$\equiv$

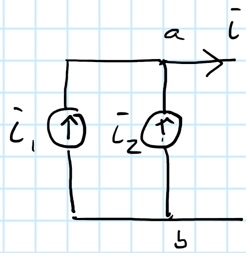


$V_{eq} = V_1 + V_2$

– Two current sources:



$\Rightarrow \boxed{i_1 = i_2} = i$

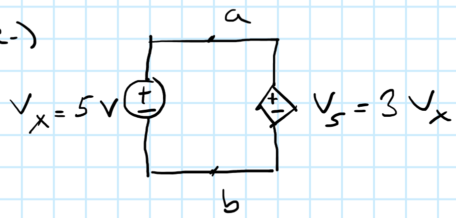


$\Rightarrow i = i_1 + i_2$

Ex:

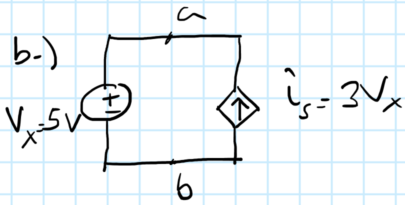
Determine which connections are valid.

a-)



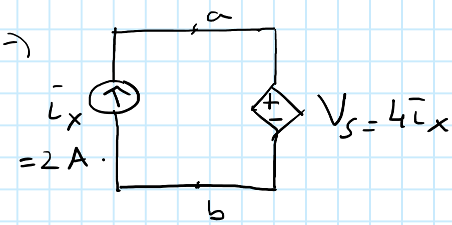
$\Rightarrow$ Not valid.

b-)



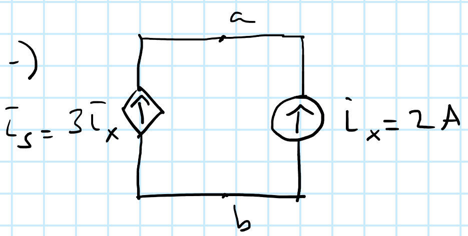
$\Rightarrow$ Ideal voltage source supplies the same voltage regardless of the current, and visa versa. Thus, this is valid.

c-)



$\Rightarrow$ Valid. Because of the same reason in part b.

d-)



$\Rightarrow$ Not valid. $i_x \neq i_s$

P3

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Two important concepts:

– Active element: A device capable of generating electrical energy. ($P<0$)

– Passive element: A device that can not generate electrical energy. ($P>0$). Examples: Resistors, inductors, capacitors. (lumped elements).

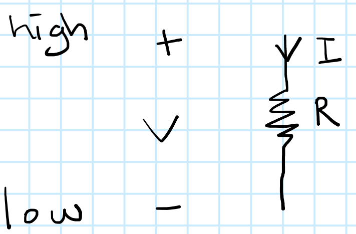
Resistance: ("R")

is the capacity of materials to resist (impede, oppose) the current flow.

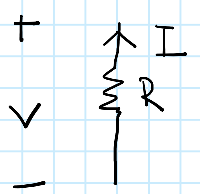
Ohm's law: $\boxed{V = I \cdot R}$

Reciprocal of resistance is called "conductance" with a symbol "G".

$\Rightarrow$ $G = \frac{1}{R}$ (Siemens (S) or ℧) ↳ "mho" R (Ω = ohm)



$\boxed{V = IR}$



$\boxed{V = -IR}$

$\boxed{G = \frac{1}{R} \cdot (S)}$

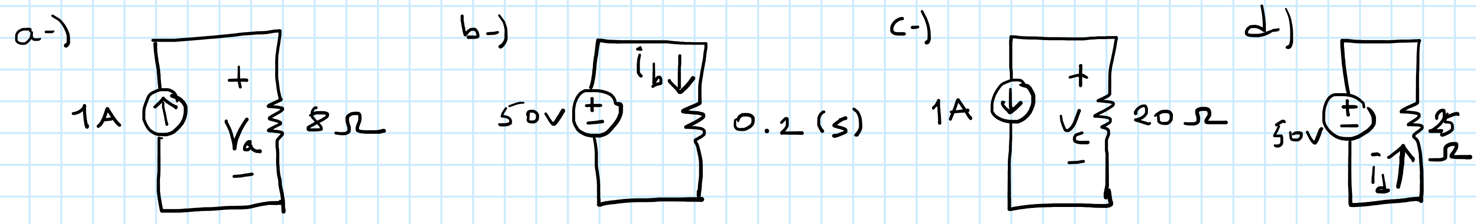
$\boxed{P = VI = I^2R \ (W).}$

Also, $\boxed{P = \frac{V^2}{R} \ (W)}$

$\boxed{I = VG}$

Ex:

Calculate the values of v and i, and determine the power dissipated in each resistor.



Ans:

a-) $V_a = i \cdot R = (1A)(8\Omega) = 8V.$, $P_{8\Omega} = \frac{V^2}{R} = \frac{8^2}{8} = 8$ Watts.

b-) $i_b = \frac{V}{R} = V \cdot G = (50V)(0.2S) = 10$ A. , $P_{0.2S} = V^2 G = 500$ W.

P3a

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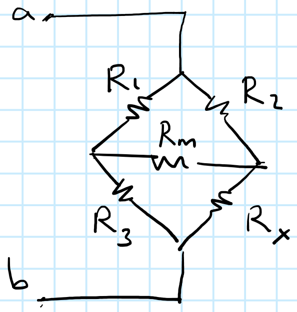
Delta to Wye Equivalent Circuits:

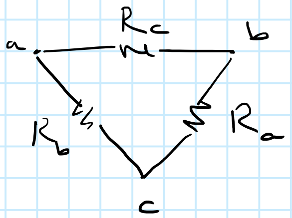
Consider the following circuit:



⇒ The resistors R_1, R_2, R_m or R_3, R_m, R_x are called "Delta (Δ)" connection.

⇒ The resistors R_1, R_m, R_3 or R_2, R_m, R_x are called "Wye (Y)" connection.

Δ to Y Transformation:



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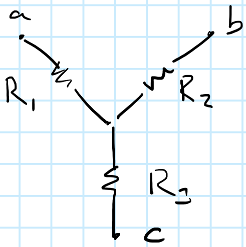


where

$$R_1 = \frac{R_b \cdot R_c}{R_a + R_b + R_c}, \quad R_2 = \frac{R_a \cdot R_c}{R_a + R_b + R_c},$$

$$R_3 = \frac{R_a \cdot R_b}{R_a + R_b + R_c}.$$

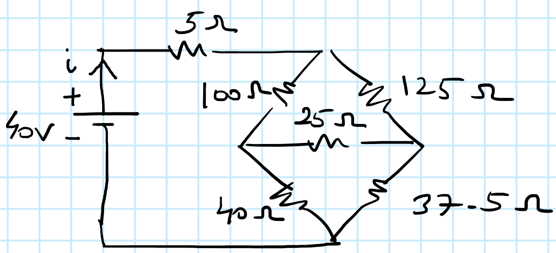
Also,

$$R_a = \frac{R_1R_2 + R_2R_3 + R_1R_3}{R_1}, \quad R_b = \frac{R_1R_2 + R_2R_3 + R_1R_3}{R_2},$$

$$R_c = \frac{R_1R_2 + R_2R_3 + R_1R_3}{R_3}.$$

Ex:

Find the current and power supplied by the 40V source.

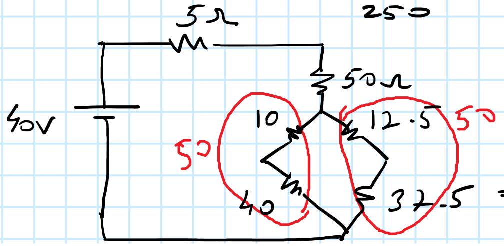


$$R_{eq} = \frac{1}{\frac{1}{50} + \frac{1}{50}} = \frac{1}{\frac{100}{2500}} = \frac{\cancel{2500}}{\cancel{100}} = 25\,\Omega$$

Ans:

First, replace the upper delta circuit (100Ω, 125Ω, 25Ω) by its Y-equivalent.

$$\Rightarrow R_1 = \frac{100 \cdot 125}{250} = 50\,\Omega, \quad R_2 = \frac{125 \cdot 25}{250} = 12.5\,\Omega, \quad R_3 = \frac{100 \cdot 25}{250} = 10\,\Omega.$$



$$\Rightarrow R_{eq} = 5 + 50 + (10 + 40) \parallel (12.5 + 37.5)$$ in parallel.

$$= 55 + \underbrace{(50 \parallel 50)}_{25} = 80\,\Omega.$$

$$\Rightarrow P = V \cdot i = -40 \cdot 0.5 = -20\,W. \qquad \Rightarrow I = V/R = \frac{40}{80} = 0.5\,A$$

$\Rightarrow P = V \cdot i = -40 \cdot 0.5 = -20W.$

25

$\Rightarrow I = V/R = \frac{40}{80} = \underline{0.5A}$

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c-) $V_c = i.R = \underbrace{-(1A)}_{i}(20\,\Omega) = \underbrace{-20V}_{?}.$, $P_{20\Omega} = \frac{V^2}{R} = \frac{(-20)^2}{20} = 20W.$

d-) $i_d = \frac{V}{R} = \frac{-50V}{25\Omega} = -2A.$, $P = \frac{V^2}{R} = (-50)^2/25 = 100W.$

"−" sign indicates that the original selection of the current direction is really the opposite.

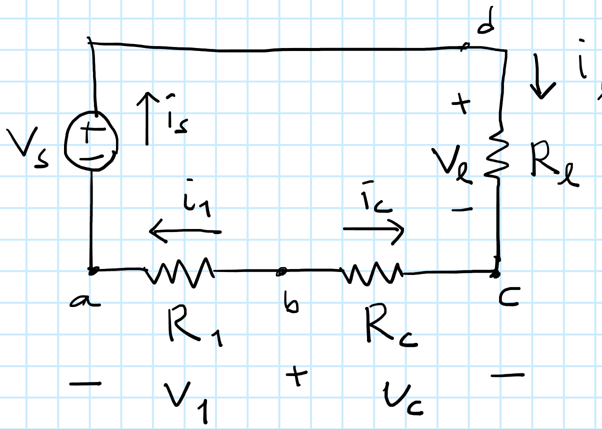
$V = -i_d(25)$

or

$50 = -i_d(25) \Rightarrow i_d = -2A$

Kirchoff's Laws:

Suppose we have the following circuit,



$V_1 = i_1 R_1$

$V_c = i_c R_c$

$V_\ell = i_\ell R_\ell$

We consider leaving currents positive, and entering currents negative.

Kirchoff's Current Law (KCL):

The algebraic sum of the currents at any point in a circuit is zero.

At point a:

$i_s - i_1 = 0$

At point b:

$i_1 + i_c = 0$

At point c:

$-i_c - i_\ell = 0$

At point d:

$i_\ell - i_s = 0$

Note that initial assumption of currents and voltages are not important. At the end of the solution, we get results which are + or −, indicating the real directions.

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Kirchoff's Voltage Law (KVL):

The algebraic sum of all the voltages around any closed path in a circuit is zero.

Closed path: dcbad (we follow clockwise direction.)

$v_\ell - v_c + v_1 - v_s = 0$ (KVL equation.)

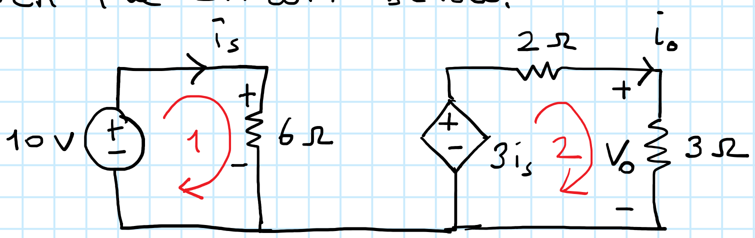
($+ \to -$ is always positive.)

Closed path: dabcd (ccw)

$v_s - v_1 + v_c - v_\ell = 0$ (The same equation as before.)

Ex:

Given the circuit below:



a-) Find V_o from KVL and Ohm's law.

b-) Show that $P_{total}\Big|_{dissipated} = P_{total}\Big|_{developed}$ (Conservation of energy.)

Ans:

a-) KVL in loop 1:

$-10V + 6\,i_s = 0$

$\Rightarrow i_s = \frac{10}{6} = \frac{5}{3}$ A.

Also, KVL for loop 2:

$-3i_s + 2i_o + V_o = 0$

$-5 + 2i_o + V_o = 0$

Ohm's law:

$V_o = 3\,i_o$

Thus,

$-5 + 2i_o + 3i_o = 0$

$5i_o = 5$

$\Rightarrow i_o = 1$ A.

Then, $V_o = 3i_o = 3$ V.

b-) Power for indep. voltage source:

$P = vi = (-10v) \cdot i_s = 10 \cdot (\frac{5}{3}) = -16.7$ W

Also,

$P_{6\Omega} = vi = (10v)\,i_s = 16.7$ W.

$P_{2\Omega} = i^2R = i_o^2(2) = (1)^2(2) = 2$ W.

$P_{3\Omega} = i^2R = i_o^2(3) = (1)^2(3) = 3$ W.

Power delivered by the dependent source:

$P = v \cdot i = (-3 i_s)(i_o) = -3 \cdot \frac{5}{3} \cdot (1A) = -5W.$

P6

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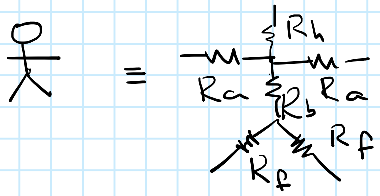
Electric Shock:

Electric current flow through the human body.
In many cases, the current flows from head towards feet passing through heart. This causes heart to stop beating (fibrilation).

Majority of 220V contacts, electric shocks result in fibrilation. This also depends on the time of exposure and the body resistence.



$\Rightarrow R_h + R_a + R_b + R_f \approx 1000\Omega$.

Dangerous limits of currents:

For AC current: 3-5mA amplitude → barely acceptable.
Amplitude > 30mA is the limit. and always risk of life.

For DC current: $I > 300$-500mA is the limit for danger of life.

Ex:

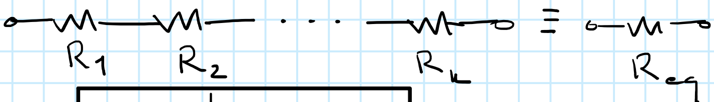
A person with a dry body gets electric shock with 220V. Find the current passing through his body?

Ans:

$$I = \frac{V}{R} = \frac{220}{1000} = 0.22\,A = 220\,mA. > 30\,mA \quad (\text{dangerous})$$

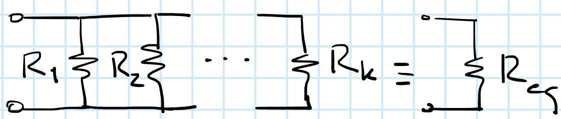
Resistive Circuits:

Resistors in Series:



$$\boxed{R_{eq} = \sum_{i=1}^{k} R_i \quad (\Omega)}$$

Resistors in Parallel:



$$\boxed{R_{eq} = \frac{1}{\sum_{i=1}^{k}\left(\frac{1}{R_i}\right)} \quad (\Omega)}$$

P7

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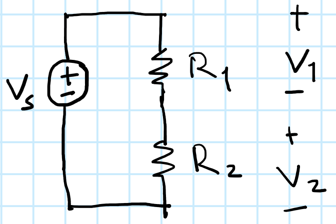
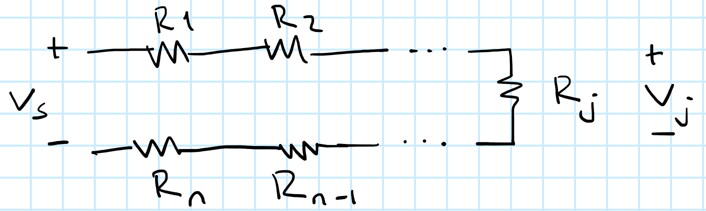
Voltage Divider:

- Short method to find voltages when several resistors are connected in series.



$$\boxed{V_1 = V_s \cdot \frac{R_1}{R_1+R_2}} \qquad \boxed{V_2 = V_s \cdot \frac{R_2}{R_1+R_2}}$$

In case of n resistors connected in series:

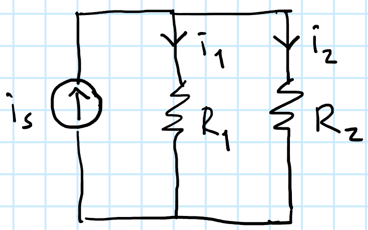


$$\boxed{V_j = V_s \cdot \frac{R_j}{R_{eq}}},$$

$$R_{eq} = R_1 + R_2 + \dots + R_n$$

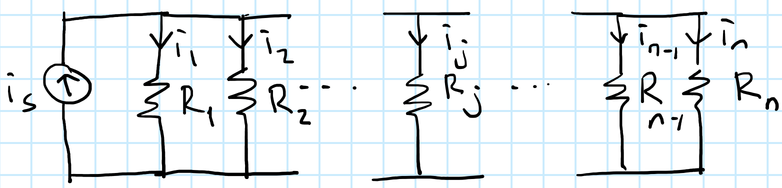
Current Divider:

It is a rule for finding the current passing through the resistors which are connected in parallel.



$$\boxed{i_1 = i_s \cdot \frac{R_2}{R_1+R_2}}, \qquad \boxed{i_2 = i_s \cdot \frac{R_1}{R_1+R_2}}$$

For general current division, we have:



$$\boxed{i_j = i_s \cdot \frac{R_{eq}}{R_j}}$$

where

$$R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_n}}$$

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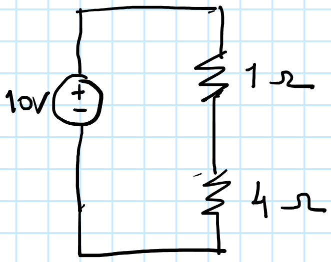
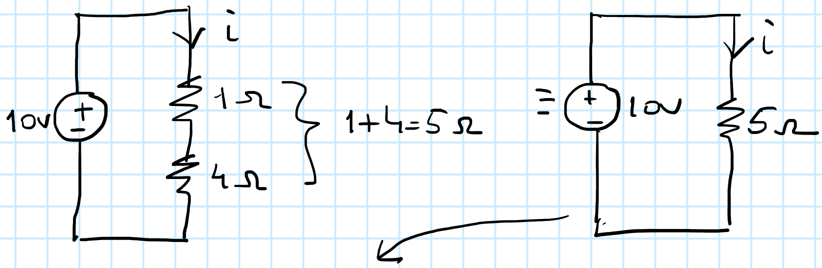
Ex:



+ $V_1 = ?$ −

+ $V_2 = ?$ −

Ans:



$$i = \frac{V}{R} = \frac{10}{5} = 2A.$$

$$V_1 = i \cdot R_1 = (2)(1) = 2\ V.$$

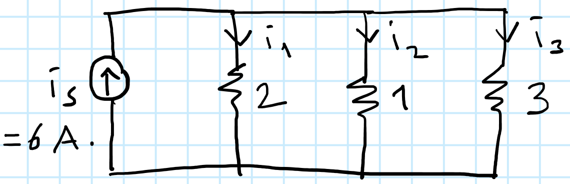
$$V_2 = i \cdot R_2 = (2)(4) = 8V$$

By using the voltage division rule:

$$V_1 = 10 \cdot \frac{1}{1+4} = \frac{10}{5} = 2V.$$

$$V_2 = 10 \cdot \frac{4}{1+4} = \cancel{10}^{2} \cdot \frac{4}{\cancel{5}} = 8V.$$

Ex:



Find i_1, i_2 and i_3 ?

Ans:

$$R_{eq} = \frac{1}{\frac{1}{2} + \frac{1}{1} + \frac{1}{3}} = \frac{1}{\frac{3+6+2}{6}} = \frac{6}{11}\ \Omega.$$

(3) (6) (2)

$$i_1 = i_s \cdot \frac{R_{eq}}{R_j} = 6 \cdot \frac{\cancel{6}^{3}}{11} \cdot \frac{1}{\cancel{2}} = \frac{18}{11} = 1.636\ A.$$

$$i_2 = 6 \cdot \frac{6}{11} \cdot \frac{1}{1} = \frac{36}{11} = 3.273$$

$$i_3 = 6 \cdot \frac{\cancel{6}^{2}}{11} \cdot \frac{1}{\cancel{3}} = \frac{12}{11} = 1.1$$

$$i_1 + i_2 + i_3 = 1.636 + 3.273 + 1.1 = 6A = i_s.\ (KCL)$$ ✓

IF we had:

$$\Rightarrow i_1 = i_s \cdot \frac{2}{1+2} = \cancel{6}^{2} \cdot \frac{2}{\cancel{3}} = 4A.$$

$$i_2 = \cancel{6}^{2} \cdot \frac{1}{\cancel{3}} = 2A.$$

P9

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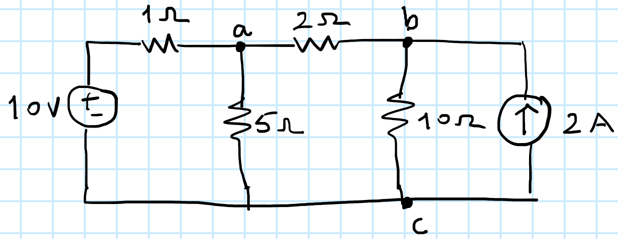
Cp. 4. Techniques of Circuit Analysis:

In a given circuit with some nodes and closed loops, we use the KCL, KVL and Ohm's law to derive algebraic equations. Then, we solve these equations simultaneously to find the unknown voltages and/or currents.

1-) Node-Voltage Method:

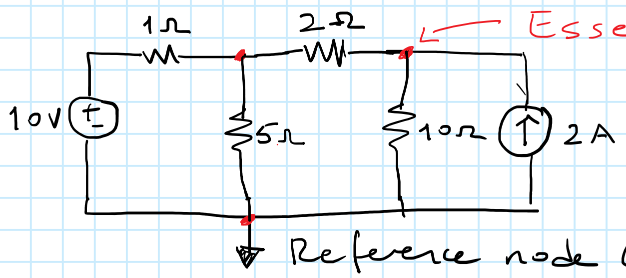
Suppose we have the following circuit:



Step 1: Simplify the circuit such that no branches cross over and we can mark the essential nodes. (3 nodes in this case.)

Step 2:

Select one of the nodes as a reference node. The node with most branches is usually taken as the reference node. (In this case, it is the lower node.)

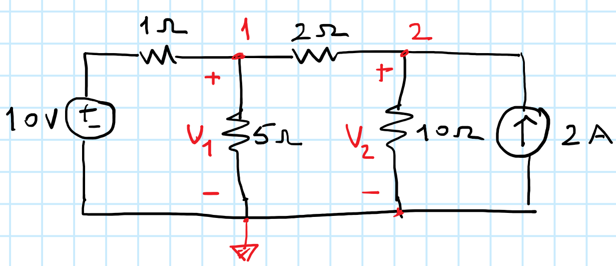


Essential nodes.

Reference node (Ground)

Step 3:

Define node voltages $\overbrace{\text{with respect to}}^{\text{w.r.t.}}$ the reference node. Thus, we have



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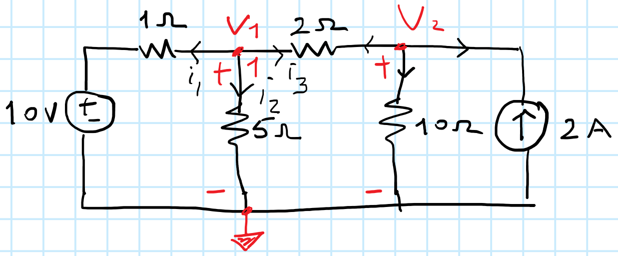
Step 4:

Generate node-voltage equations by:
Writing the node equations for each node employing the leaving current convention.



KCL at node 1 gives:

$$i_1 + i_2 + i_3 = 0$$

$$\frac{V_1 - 10}{1\Omega} + \frac{V_1}{5\Omega} + \frac{V_1 - V_2}{2\Omega} = 0 \qquad (1)$$

KCL at node 2 gives:

$$\frac{V_2 - V_1}{2\Omega} + \frac{V_2}{10\,\Omega} - 2 = 0 \qquad (2)$$

Equations (1) and (2) can be solved simultaneously to find

$V_1 = 9.09V$, $\quad V_2 = 10.91V$.

The currents can be found by Ohm's law.

For example $i_3 = \frac{V_1 - V_2}{2} = \frac{9.09 - 10.91}{2} = -0.91$

- The minus sign indicates that the real direction of the current is opposite of our initial selection.
- Voltage polarities and direction of currents can be arbitrarily selected in the beginning. The important point is that we must be consistent with our initial selections throughout the solution.

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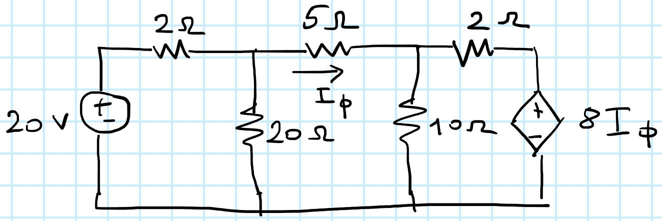
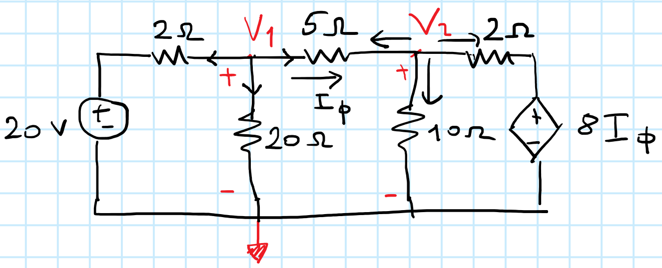
Ex:

Given the following circuit



Find $P_{5\Omega} = ?$

Ans:



$$I_\phi = \frac{V_1 - V_2}{5} \text{ from the ohm's law.} \quad \text{———} (1)$$

Use the node-voltage method:

KCL at node 1:

$$\frac{V_1 - 20}{2} + \frac{V_1}{20} + I_\phi = 0 \quad \text{———} (2)$$

KCL at node 2:

$$-I_\phi + \frac{V_2}{10} + \frac{V_2 - 8I_\phi}{2} = 0 \quad \text{———} (3)$$

Thus, we have 3 equations with 3 unknowns which can be solved simultaneously.

$\Rightarrow V_1 = 16V, \quad V_2 = 10V, \quad I_\phi = 1.2A.$

$P_{5\Omega} = I_\phi^2 \cdot R = (1.2)^2 (5) = 7.2W$ //

2-) Mesh-Current Method:

Mesh → loop with no other loops inside.

For example, consider the following circuit:

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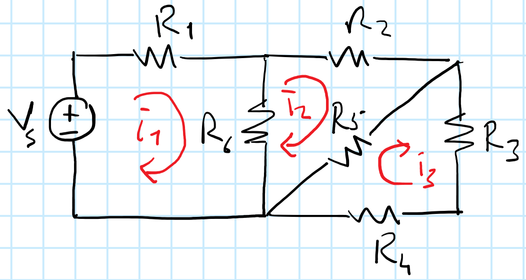


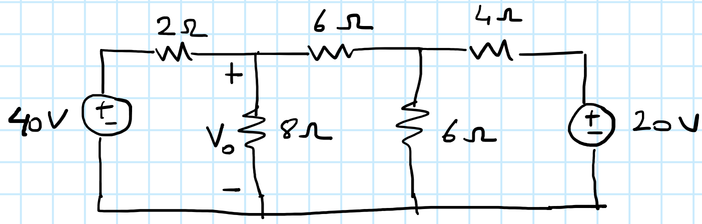
How many meshes are there in this circuit?

Ans: There are 3 meshes with currents i_1, i_2 and i_3.

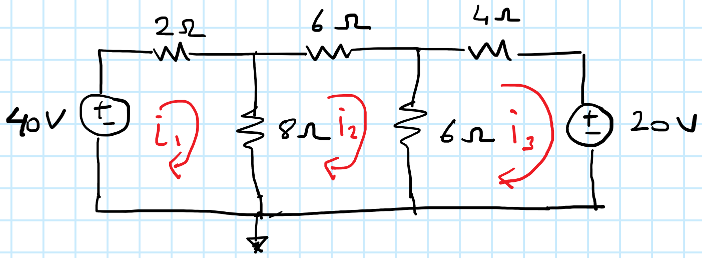
For mesh-current analysis, we write the KVL equations for each mesh, and solve the equations simultaneously.

Ex:

Use the mesh-current method to determine the power associated with each voltage source. Also find V_o.



Ans:



KVL for mesh 1:

$$-40V + 2i_1 + 8(i_1 - i_2) = 0 \quad \text{——} (1)$$

KVL for mesh 2:

$$8(i_2 - i_1) + 6i_2 + 6(i_2 - i_3) = 0 \quad \text{——} (2)$$

KVL for mesh 3:

$$6(i_3 - i_2) + 4i_3 + 20 = 0 \quad \text{——} (3)$$

Again, we have 3 equations with 3 unknowns (i_1, i_2, i_3).

Re-arranging the equations:

$$10i_1 - 8i_2 = 40$$
$$-8i_1 + 20i_2 - 6i_3 = 0$$
$$-6i_2 + 10i_3 = -20$$

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$$10\,i_1 - 8\,i_2 = 40$$
$$-8\,i_1 + 20\,i_2 - 6\,i_3 = 0$$
$$-6\,i_2 + 10\,i_3 = -20$$

$$\underbrace{\begin{bmatrix} 10 & -8 & 0 \\ -8 & 20 & -6 \\ 0 & -6 & 10 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix}}_{I} = \underbrace{\begin{bmatrix} 40 \\ 0 \\ -20 \end{bmatrix}}_{b} \Rightarrow A.I = b \text{ (matrix equation)}$$

$$A \,.\, I = b$$

The solution is:

$$\boxed{I = A^{-1}.\,b}$$

$i_1 = 5.6\,A$, $i_2 = 2\,A$, $i_3 = -0.8\,A$.

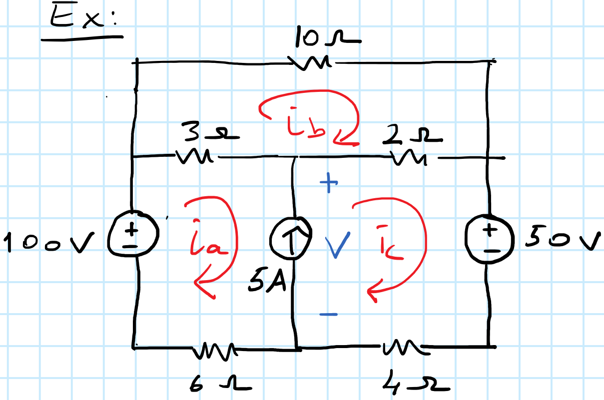
$$\left.\begin{aligned} P_{40V} &= -40\,.\,i_1 = -224\,W. \\ P_{20V} &= -20\,.\,i_3 = -16\,W. \end{aligned}\right\} \text{Power delivered.}$$

$$V_o = 8(i_1 - i_2) = 8(3.6) \Rightarrow V_o = 28.8\,V //.$$

Special Case of Mesh-Current Method:

When there is a current source in the circuit, the number of unknown currents is reduced by one.

Ex:



Find the loop currents i_a, i_b and i_c in this circuit.

Ans:

For mesh a: (KVL)

$$-100 + 3(i_a - i_b) + V + 6\,i_a = 0 \quad \text{—— (1)}$$

For mesh c:

$$50 + 4\,i_c - V + 2(i_c - i_b) = 0 \quad \text{—— (2)}$$

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Re-write eqn's (1) and (2) as

$$100 = 3(i_a - i_b) + V + 6i_a \quad \text{——(1)}$$

$$-50 = 4i_c - V + 2(i_c - i_b) \quad \text{——(2)}$$

Add (1) and (2)

$$50 = 9i_a - 5i_b + 6i_c \quad \text{——(3)}$$

<u>KVL for mesh b:</u>

$$0 = 3(i_b - i_a) + 10i_b + 2(i_b - i_c) \quad \text{——(4)}$$

We also have

$$i_c - i_a = 5A. \quad \text{——(5)}$$

Eqn's (3), (4) and (5) can be solved for i_a, i_b and i_c.

$$\underbrace{\begin{bmatrix} 9 & -5 & 6 \\ -3 & 15 & -2 \\ -1 & 0 & 1 \end{bmatrix}}_{A} \cdot \underbrace{\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}}_{i} = \underbrace{\begin{bmatrix} 50 \\ 0 \\ 5 \end{bmatrix}}_{b}$$

$$9i_a - 5i_b + 6i_c = 50 \quad \text{——(3)}$$

$$-3i_a + 15i_b - 2i_c = 0$$

$$-i_a + i_c = 5$$

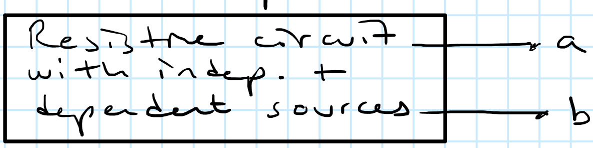
$$\Rightarrow i = A^{-1} \cdot b$$

$$\Rightarrow i_a = 1.75A, \quad i_b = 1.25A, \quad i_c = 6.75A.$$

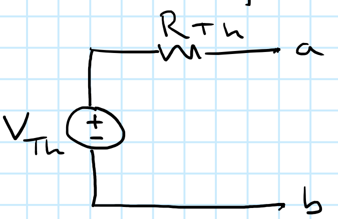
Chapter 5:

<u>Thévenin and Norton Equivalents:</u>

We want to replace

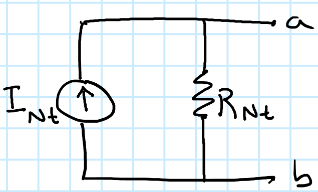


with the following circuit:



where V_{Th} = Thévenin voltage.

R_{Th} = Thévenin resistance.



where I_{Nt} = Norton current

R_{Nt} = Norton resistance

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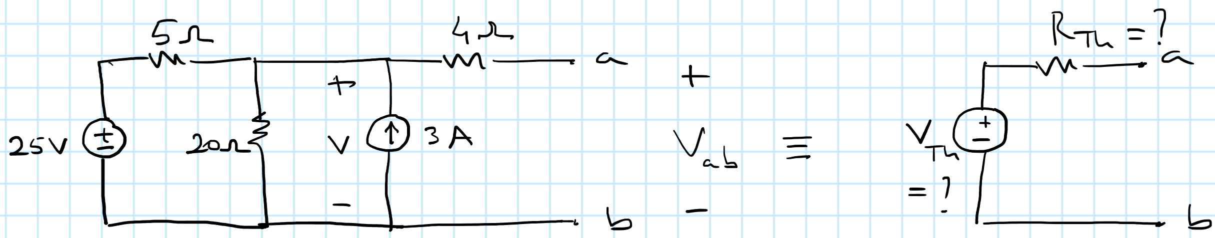
Consider the following circuit:

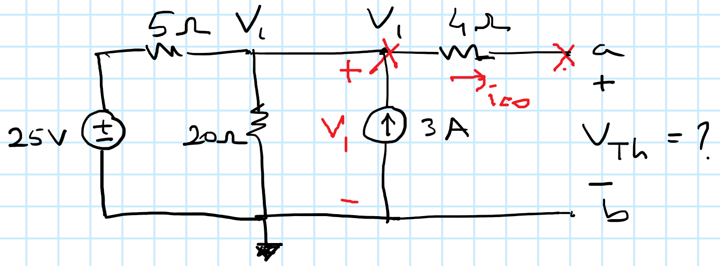


– If we connect another circuit to the given circuit at points a and b, this other circuit sees no difference if it is connected to Thévenin equivalent circuit.

V_{Th} = Open circuit voltage btw the terminals a and b, and we short circuit a and b, and find the current btw a and b (i_{sc}).

Thus, $i_{sc} = \dfrac{V_{Th}}{R_{Th}} \Rightarrow \boxed{R_{Th} = \dfrac{V_{Th}}{i_{sc}}}$

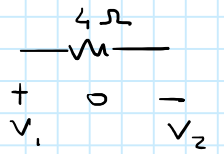
For the given circuit; using the node-voltage method:



The current passing through the 4Ω resistor is zero.

$V = i \cdot R = 0$

$i = 0$



$V_1 - V_2 = 0$

$\boxed{V_1 = V_2}$

$\Rightarrow \boxed{V_1 = V_{Th}}$.

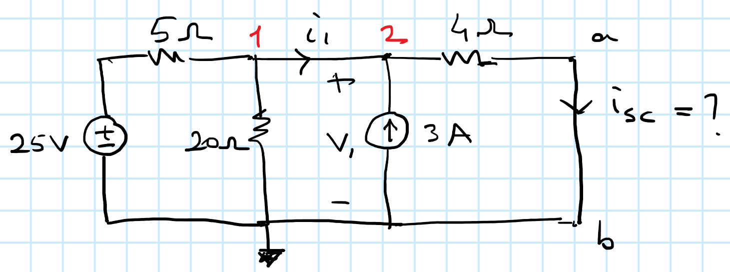
KCL at node 1:

$$\frac{V_1 - 25}{5} + \frac{V_1}{20} - 3 = 0$$

$\Rightarrow V_1 = 32V.$

$V_{Th} = 32V.$

– In order to find R_{Th}, we short circuit points a and b, and find i_{sc}.



KCL at node 1:

$$\frac{V_1 - 25}{5} + \frac{V_1}{20} + i_1 = 0 \quad \text{—— (1)}$$

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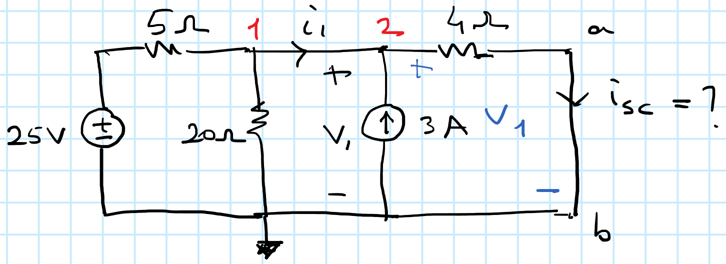


KCL at node 2:

$$-i_1 - 3\,A + i_{sc} = 0$$

or

$$i_{sc} - i_1 = 3\,A \quad \text{where } i_{sc} = \frac{V_1}{4} \Rightarrow \frac{V_1}{4} - i_1 = 3 \quad \text{—}(2)$$

Earlier, we had

$$\frac{V_1 - 25}{5} + \frac{V_1}{20} + i_1 = 0 \quad \text{—}(1)$$

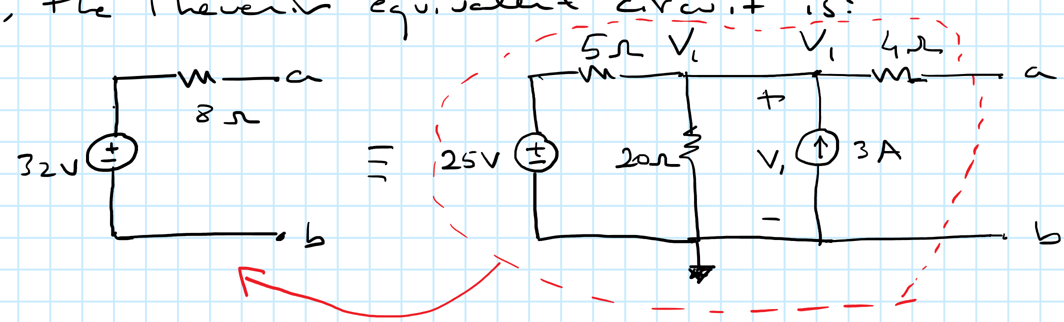
Thus, (1) and (2) can be solved simultaneously and

$$\Rightarrow V_1 = 16\,V.$$

$$\Rightarrow i_{sc} = \frac{V_1}{4} = \frac{16}{4} = 4\,A.$$

$$\Rightarrow R_{Th} = \frac{V_{Th}}{i_{sc}} = \frac{32\,V}{4\,A} = 8\,\Omega.$$

Thus, the Thévenin equivalent circuit is:



In order to obtain the "Norton Equivalent Circuit", we can make "source transformation", where $R_{Th}\big|_{\text{Thevenin}} = R_{th}\big|_{\text{Norton}}$

The current source of Norton circuit is the short circuit current, that is I_{sc}.

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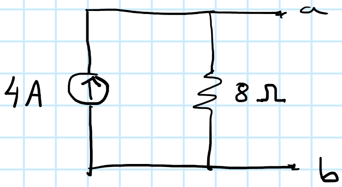
Then, for this example, the Norton equivalent circuit is



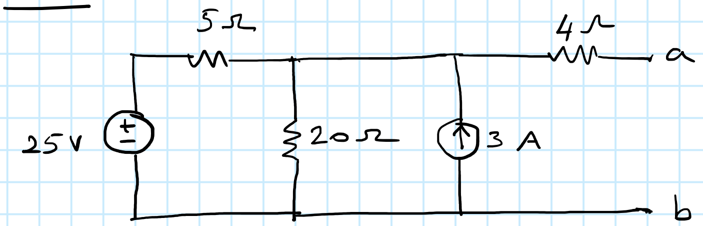
(Norton equivalent).

Alternative technique to find Thévenin equivalent resistance:

If there are only independent sources, to find the Thévenin equivalent resistance:

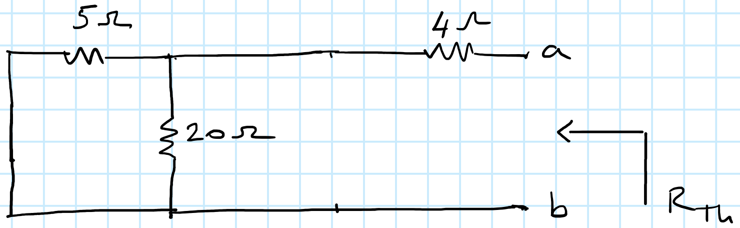
- Short circuit voltage sources and } Deactivating the
- Open circuit current sources. } indep sources
- Then, evaluate the resistance R_{Th} wrt a and b.

Ex:



Find $R_{Th} = ?$ w.r.t points a and b.

Ans:



$$\Rightarrow R_{Th} = 4 + (5 \parallel 20)$$

$$= 4 + \frac{20 \cdot 5}{20+5} = 4 + \frac{\cancel{20} \cdot 5}{\cancel{25}}$$

$$= 4 + 4 = 8\ \Omega.$$

Another alternative technique to find Thévenin resistance:

If the circuit contains dependent sources:

- First, deactivate all indep. sources
- Apply a test voltage "V_T" btw. terminals a and b.
- $R_{Th} = \dfrac{V_T}{i_T}$, where i_T is the current passing through V_T.

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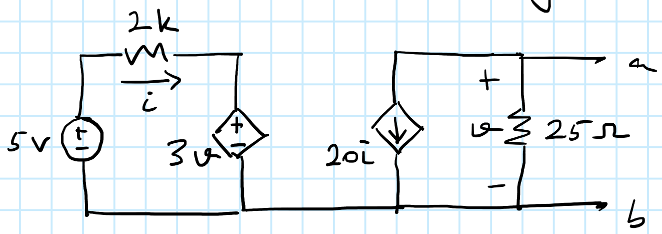
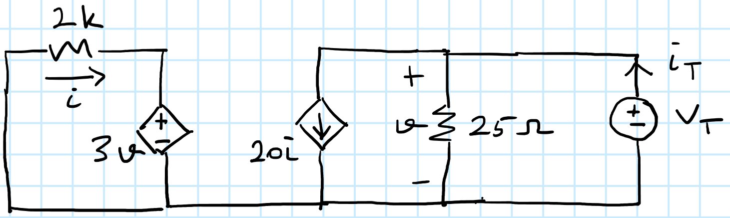
Ex:

Find the Thévenin resistance R_{Th} for the circuit below using the method that was just described.



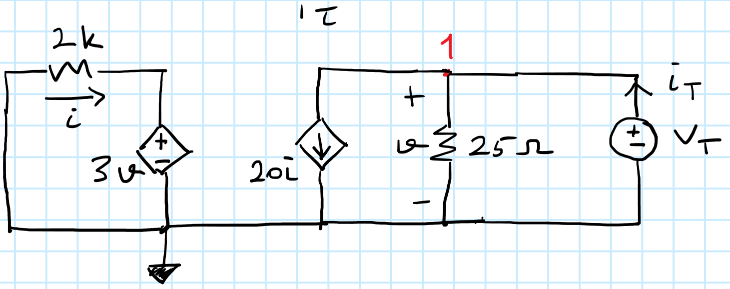
Ans:

– First, we de-activate all indep. sources:



– Apply the test voltage V_T

– We need to find $\frac{V_T}{i_T} = R_{Th}$. Let us use the node-voltage method:



Prefix	Value
T (tera)	10^{12}
G (giga)	10^{9}
M (mega)	10^{6}
k (kilo)	10^{3}
c (centi)	10^{-2}
m (milli)	10^{-3}
μ (micro)	10^{-6}
n (nano)	10^{-9}
p (pico)	10^{-12}
f (femto)	10^{-15}

KCL at node 1:

$$20i + \frac{v}{25} = i_T \quad \text{——(1)}$$

where $i = \frac{-3v}{2k} = \frac{-3V_T}{2000}$ (since $v = V_T$)

Then,

$$20 \cdot \left(\frac{-3V_T}{2000}\right) + \frac{V_T}{25} = i_T \quad \text{——(2)}$$

Solve equation (2) for $\frac{V_T}{i_T}$.

$$\frac{i_T}{V_T} = \frac{1}{100} \Rightarrow R_{Th} = \frac{V_T}{i_T} = 100\Omega \,//$$

$$\underset{(1)}{\frac{-3V_T}{100}} + \underset{(4)}{\frac{V_T}{25}} = i_T$$

$$\frac{-3V_T + 4V_T}{100} = i_T$$

$$\frac{V_T}{100} = i_T$$

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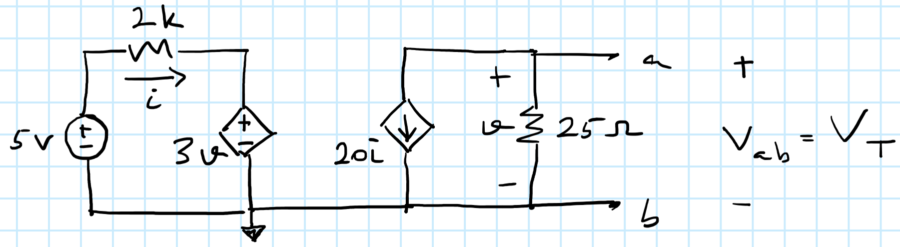
– If V_T is asked to be found, then



$\Rightarrow V_{Th} = V_{ab} = (-20i)(25) = -500i$ and

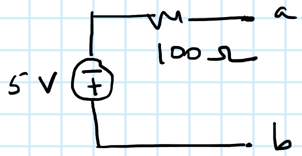
from the 1st circuit loop,

$$i = \frac{5-3v}{2000} = \frac{5-3v_{ab}}{2000} = \frac{5-3(-500i)}{2000}$$

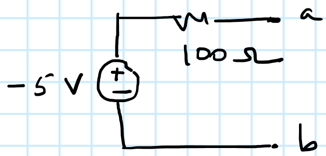
$\Rightarrow 20\cancel{00}\,i = 5 + 15\cancel{00}\,i \Rightarrow 500i = 5 \Rightarrow i = 0.01\,A = 10\,mA.$

$\Rightarrow V_{Th} = -500\,(10\,mA) = -500(0.01) = -5\,V.$

Thus, the Thévenin Eq. circuit is:

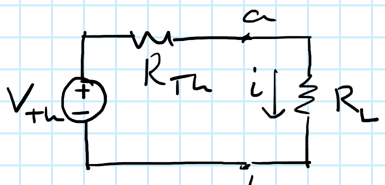


or



Maximum Power Transfer:

Consider the following circuit:



– The power consumed by the resistor R_L is:

$$P_{R_L} = i^2 \cdot R_L \ (W)$$

– We want to find a value for R_L such that P_{R_L} is maximum. This is called "maximum power transfer".

– The power consumed by R_L, P_{R_L}, is a function of R_L.

$$\Rightarrow P_{R_L} = P_{R_L}(R_L) = i^2 R_L = \left(\frac{V_{Th}}{R_{Th} + R_L}\right)^2 \cdot R_L$$

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$$P_{R_L} = P_{R_L}(R_L) = i^2 R_L = \left(\frac{V_{Th}}{R_{Th}+R_L}\right)^2 \cdot R_L$$

To find the max. $P_{R_L}(R_L)$

$$\frac{d\, P_{R_L}(R_L)}{d\, R_L} = 0 .$$

$$\Rightarrow\ 2\left(\frac{V_{Th}}{R_{Th}+R_L}\right)\cdot\left[\frac{-V_{Th}}{(R_{Th}+R_L)^2}\right] R_L + \left(\frac{V_{Th}}{R_{Th}+R_L}\right)^2 = 0$$

Re-arange the equation,

$$2\cancel{\left(\frac{V_{Th}}{R_{Th}+R_L}\right)^2}\cdot\frac{1}{(R_{Th}+R_L)}\cdot R_L = \cancel{\left(\frac{V_{Th}}{R_{Th}+R_L}\right)^2}$$

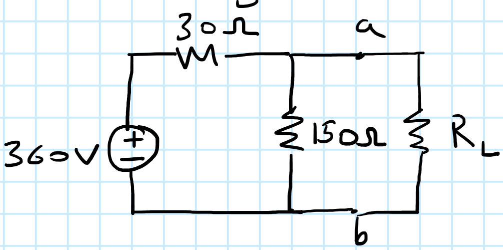
$$2\cancel{R}_L = R_{Th} + \cancel{R}_L$$

$$\Rightarrow\ \boxed{R_L = R_{Th}}$$ (Condition for max. power transfer).

$$\boxed{P_{R_L} = \left(\frac{V_{Th}}{R_{Th}+R_L}\right)^2 \cdot R_L = \frac{V_{Th}^2}{4R_L^{\cancel{2}}}\cdot\cancel{R_L} = \frac{V_{Th}^2}{4R_L}\ (W)}$$

Ex:

For the given circuit below:

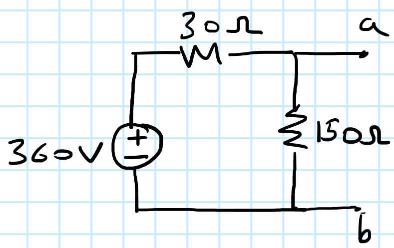


i) Find R_L for max. power transfer

ii) Find $P_{R_L}\Big|_{max}$.

Ans:

- First, we find the Thévenin equivalent circuit for the left side of the terminals a and b.



≡ Thévenin equivalent = ?

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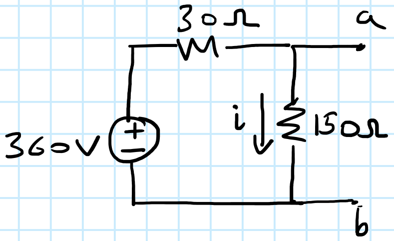


i) $V_{Th} = 360 \cdot \frac{150}{150+30} = 360 \cdot \frac{150}{180} = 300V.$

$R_{Th} = 30\Omega \,||\, 150\Omega = \frac{150 \cdot 30}{150+30} = \frac{150 \cdot \cancel{30}}{1\cancel{8}0} = 25\Omega.$

$R_L = R_{Th}$ for max. power transfer $\Rightarrow R_L = 25\Omega.$

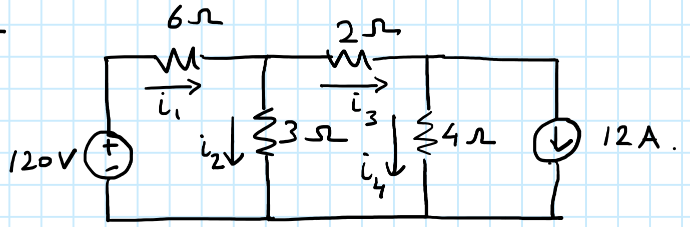
ii) $P_{R_L} = i^2 \cdot R_L = \left(\frac{V_{Th}}{R_{Th}+R_L}\right)^2 \cdot R_L = \left(\frac{300}{25+25}\right)^2 \cdot 25 \Rightarrow P_{R_L} = 900$ W.

$\frac{300^2}{4 \cdot 25} = \frac{300 \cdot 3\cancel{00}}{\cancel{100}} = 900$ W

Superposition:

Superposition is the concept of "Linearity". It means that when a system is fed by more than one independent sources, the total response is sum of individual responses.

Ex:



Find i_1, i_2, i_3 and i_4 by using the superposition.

Ans:

- This circuit has 2 indep. sources.
- De-activate one of the sources, and solve the circuit. Then de-activate the other source, and sole the circuit.
- Sum the two solutions from the previous step. Because the resistive circuits with indep. sources are linear.
- Thus, let us first de-activate the 12A source, and find the unknown currents i_1, i_2, i_3 and i_4 :

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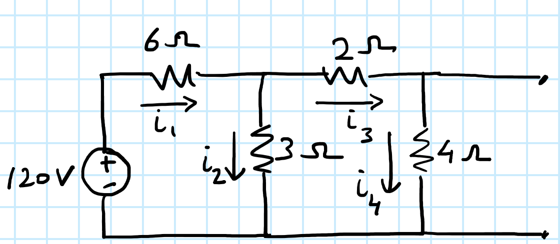


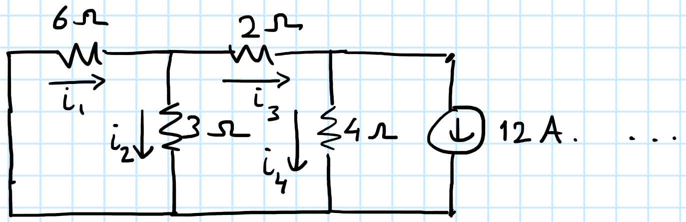
... $i_1' = 5A$

$i_2' = 10A$

$i_3' = 5A$

$i_4' = 5A$

Now, de-activate the 1st indep. source:



... $i_1'' = 2A$

$i_2'' = -4A$

$i_3'' = 6A$

$i_4'' = -6A$.

Therefore, the unknown currents are

$i_1 = i_1' + i_1'' \Rightarrow i_1 = 7A.$

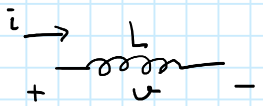
$i_2 = i_2' + i_2'' \Rightarrow i_2 = 6A.$

$i_3 = i_3' + i_3'' \Rightarrow i_3 = 11A.$

$i_4 = i_4' + i_4'' \Rightarrow i_4 = -1A.$

Chapter 6: Inductance and Capacitance:

- **Inductor:** It is an electrical element that can be a part of a wire having circular turns.



- The relation among v, i and L is:

$$\boxed{v = L \cdot \frac{di}{dt}}, \quad v = v(t), \quad i = i(t), \quad L$$

L = Inductance = constant (Henry) - conversely

$$\boxed{i(t) = \frac{1}{L}\int_{t_0}^{t} v \, d\tau + i(t_0)}$$

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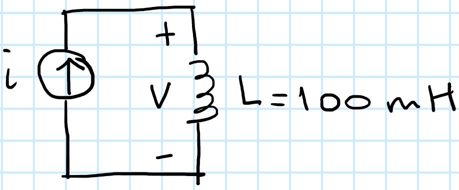
Ex:

For the given circuit:



Given that

$i = 0 \,,\; t < 0$

and

$i(t) = 10\,t\,e^{-5t}$ (A), $t \geq 0$

Find $v = ?$ for $t \geq 0$.

Ans:

$$V = L\frac{di}{dt} = (0.1)\cdot\frac{d}{dt}\,i(t) = (0.1)\,\frac{d}{dt}\left[10te^{-5t}\right]$$

or

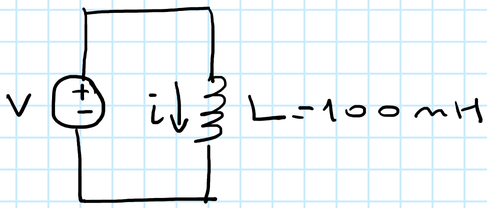
$$V = (0.1)\cdot\left[10e^{-5t} + 10t\cdot(-5e^{-5t})\right]$$

or

$$V = (0.1)(10e^{-5t})(1-5t) = e^{-5t}(1-5t)\ \text{V}\,,\ t \geq 0.$$

Ex:

For the given circuit:



Given:

$V = 20\,t\,e^{-10t}$ (V), $t > 0$

$i = 0 \,,\; t \leq 0$.

Find $i(t)$, $t > 0$.

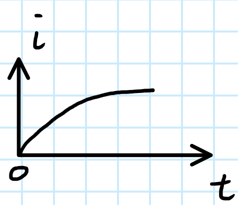
τ = tau.

Υ = tau.

Ans:

$$i(t) = \frac{1}{L}\int_{t_0}^{t} v\,d\tau + i(t_0) = \frac{1}{0.1}\cdot\int_{0}^{t} 20\tau e^{-10\tau}\,d\tau + 0.$$

$$\Rightarrow\quad i(t) = 2\left(1 - 10te^{-10t} - e^{-10t}\right)\ (\text{A}),\ t > 0.$$



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Power and Energy in an Inductor:

- For $p = i \cdot v$, substitute i and v for an inductor:

$$\boxed{p = i \cdot L \cdot \frac{di}{dt} = L\, i \frac{di}{dt}} \quad (W)$$

- The energy is

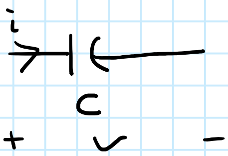
$$p = \frac{dE}{dt} \quad \text{or} \quad \frac{dE}{\cancel{dt}} = L\, i \frac{di}{\cancel{dt}} \Rightarrow \underbrace{dE}_{\text{differential energy}} = L\, i \underbrace{di}_{\text{differential current.}}$$

Take the integral of both sides:

$$\int_0^E dE = L \int_0^i i\, di \Rightarrow \int_0^E d\tau = L \int_0^i \alpha\, d\alpha$$

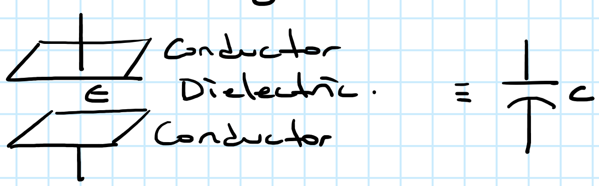
$$\Rightarrow E = L \cdot \frac{i^2}{2} \Rightarrow \boxed{E = \frac{1}{2} L\, i^2}$$ (Stored energy inside an inductor.

Capacitor:



- Capacitor is an electrical element that is made of two conductors separated by an insulator (dielectric).

C = constant = capacitance



- We have the following circuit relation for a capacitor:

$$\boxed{i = C \cdot \frac{dv}{dt}}$$

Conversely, we have

$$\boxed{v(t) = \frac{1}{C} \int_{t_0}^{t} i\, d\tau + v(t_0)}$$

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Power and Energy in a Capacitor:

$$\boxed{p = v \cdot i = C \cdot v \cdot \frac{dv}{dt} \ (W)}$$

$$\boxed{E = \frac{1}{2} C V^2 \ (J)}$$

& = & = and.

Series and Parallel Connection of Inductors & Capacitors:

Series:

For inductors, $\boxed{L_{eq} = L_1 + L_2 + \dots + L_n}$ (n inductors in series)

For capacitors, $\boxed{\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \dots + \frac{1}{C_n}}$ (n capacitors in series)

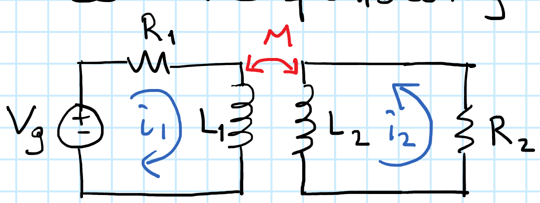
Parallel:

For inductors, $\boxed{\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \dots + \frac{1}{L_n}}$

For capacitors, $\boxed{C_{eq} = C_1 + C_2 + \dots + C_n.}$

Mutual Inductance:

Consider the following circuit

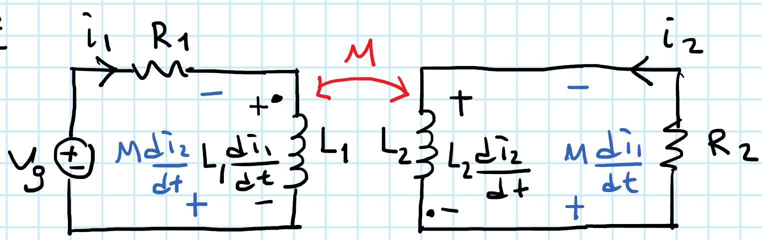


M: Mutual inductance.

Dot Conversion:

When the reference direction for a current enters the dotted terminal, the reference polarity of the voltage that is induced in the other coil is positive at its dotted terminal.

Ex:



– $M\frac{di_1}{dt}$ and $M\frac{di_2}{dt}$ are mutual induced voltage.

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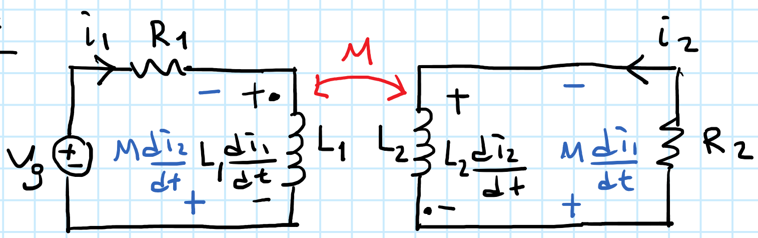
Ex:



- $M\frac{di_1}{dt}$ and $M\frac{di_2}{dt}$ are the mutual induced voltage.
- $L_1\frac{di_1}{dt}$ and $L_2\frac{di_2}{dt}$ are the self inductance voltages.

Write the KVL equations for the above circuit

Ans:

In the 1st loop:

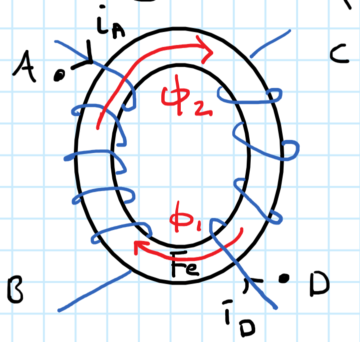
$$-V_g + i_1R_1 + L_1\frac{di_1}{dt} - M\frac{di_2}{dt} = 0 \quad \text{---(1)}$$

In the 2nd loop:

$$i_2R_2 + L_2\frac{di_2}{dt} - M\cdot\frac{di_1}{dt} = 0 \quad \text{---(2)}$$

Determining Dot Markings:

Consider the following inductor (toroid):



Steps:

1-) Arbitrarly select one terminal, and mark it with a dot. (Let's say D)

2-) Assign a current into the dotted terminal (i_D).

3-) Determine the magnetic flux by i_D using the "Right hand rule" (ϕ_1).

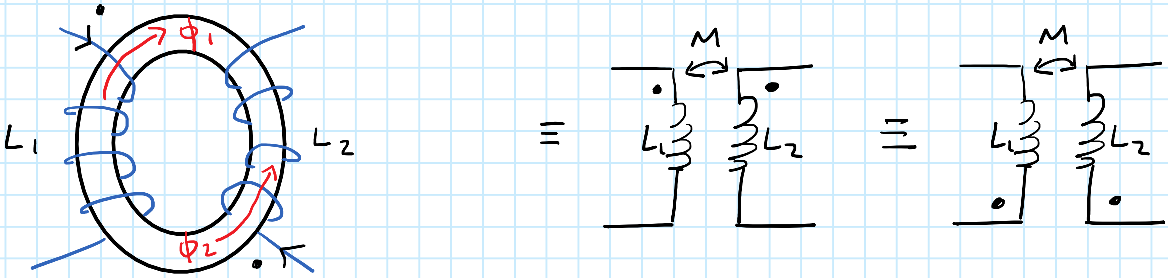
4-) Pick one terminal on the 2nd coil (Let's say A), and assign a current into this terminal (i_A).

5-) Find ϕ_2 by i_A.

6-) If ϕ_1 and ϕ_2 are in the same direction, then put a dot on the selected terminal (in this case, terminal A). If not, put a dot on the opposite terminal (terminal B).

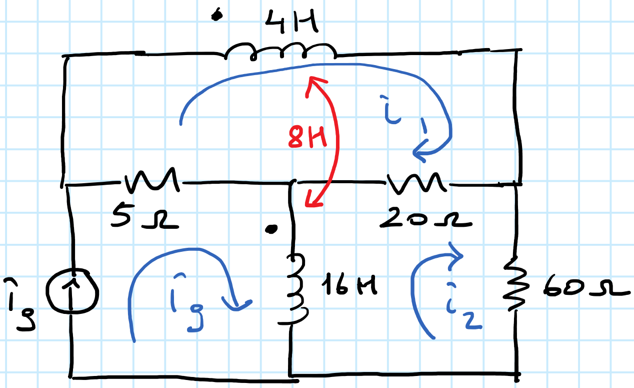
Ex:

For the given toroidal inductors find the dot convention.



Ex:

For the given circuit, write the equations for the mesh currents i_1 and i_2.



Ans:

For mesh 1:

$$4\frac{di_1}{dt} + 8\frac{d(i_g - i_2)}{dt} + 20(i_1 - i_2) + 5(i_1 - i_g) = 0 \qquad \text{—(1)}$$

For mesh 2:

$$20(i_2 - i_1) + 60 i_2 + \underbrace{16\frac{d(i_2 - i_g)}{dt}}_{\text{self induced voltage}} \overbrace{- 8\frac{di_1}{dt}}^{\text{mutual induced voltage}} = 0 \qquad \text{—(2)}$$

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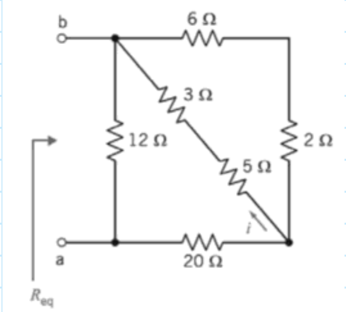
Sample Questions:

1-)

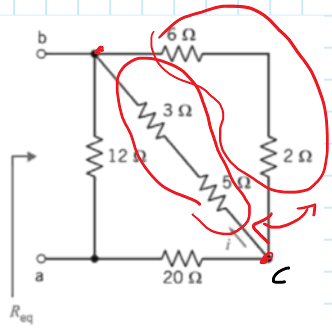


Find i and R_{eq} if $V_{ab} = 40V$

Ans:

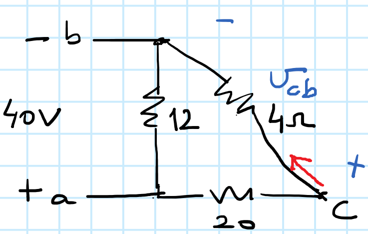
$$R_{eq} = 12 \parallel \left\{ 20 + \underbrace{\left[\underbrace{(5+3)}_{8} \parallel \underbrace{(6+2)}_{8} \right]}_{4} \right\}$$

$$\underbrace{20 + 4}_{24}$$



$$\Rightarrow R_{eq} = 12 \parallel 24 = \frac{12 \cdot 24}{12 + 24} = \frac{\cancel{12} \cdot 24}{\cancel{36}_{3}} = 8\,\Omega.$$

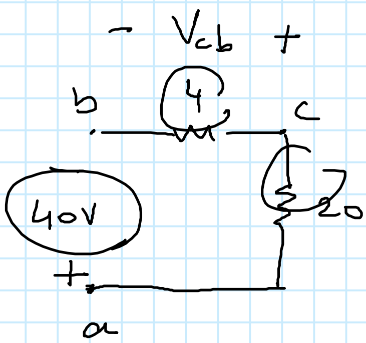
To find i,



$$V_{ab} = V_a - V_b$$

$$V_{cb} = 40 \cdot \frac{4}{20+4}$$

$$\Rightarrow V_{cb} = 40 \cdot \frac{\cancel{4}}{\cancel{24}_{6}} = \frac{40}{6}$$



Then,

$$i = \frac{V_{cb}}{R} = \frac{40/6}{8} = \frac{\cancel{40}^{5}}{6} \cdot \frac{1}{\cancel{8}} = \frac{5}{6}\,A.$$

2-) For the circuit in Fig.2, if the power delivered by the source is 20 mW, find R and V_s.

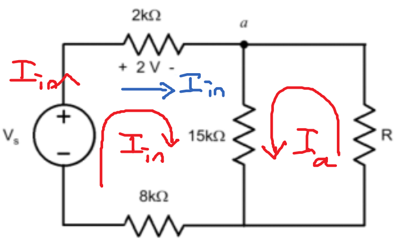


Figure 2: Circuit for question 2.

$$I_{in} = \frac{2V}{2k} = 1\,mA.$$

$$P_{V_s} = 20mW = I_{in} \cdot V_s$$

$$\Rightarrow V_s = \frac{20\,\cancel{m}W}{1\,\cancel{m}A} = 20\,V.$$

KVL in the left loop: $-20 + 2 + (I_a + I_{in})(15k) + I_{in}(8k) = 0$

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$$-20 + 2 + (I_a + I_{in})(15k) + I_{in}(8k) = 0$$

$$15k\, I_a + 23k \underbrace{I_{in}}_{1mA} = 18$$

$$\Rightarrow I_a = \frac{18-23}{15} mA = \frac{-5}{15} mA = -\frac{1}{3} mA$$

$$\Rightarrow V_a = (I_{in} + I_a)(15k)$$

$$= \left(1 - \frac{1}{3}\right) \cancel{m}A\, (15\cancel{k})$$

$$= \frac{2}{\cancel{3}} (\cancel{15})^{5} = 10V.$$

Thus, $R = \frac{V_a}{-I_a} = \frac{10}{\frac{1}{3} mA} = 30k\Omega.$

For the circuit in Fig.2, if the power delivered by the source is 20 mW, find R and V_s.

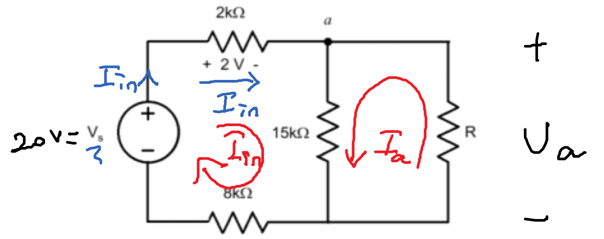

Figure 2: Circuit for question 2.

3-)

Find the voltage v_a in Fig.3 by
a-) Node-voltage analysis
b-) Source transformation.

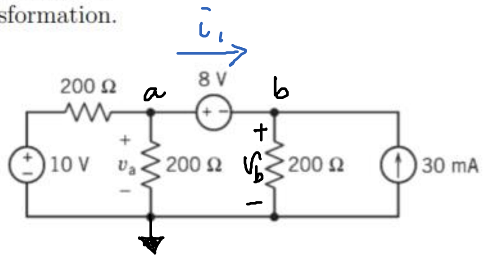

Figure 3: Circuit for question 3.

a-) KCL at node a:

$$\frac{v_a - 10}{200} + \frac{v_a}{200} + i_1 = 0 \quad \text{——(1)}$$

KCL at node b:

$$-i_1 + \frac{v_b}{200} - 30mA = 0 \quad \text{——(2)}$$

$$v_a - v_b = 8 \quad \text{——(3)}$$

Add (1) and (2):

$$\frac{v_a - 10}{200} + \frac{v_a}{200} + \frac{v_b}{200} = 30mA.$$

$$v_a - 10 + v_a + v_b = (2\cancel{00})(3\cancel{0}\, \cancel{mA}) = 6$$

$$2v_a + v_b = 16 \quad \text{——(4)}$$

Add (3) and (4):

$$v_a - \cancel{v_b} = 8$$

$$+\ 2v_a + \cancel{v_b} = 16$$

$$3v_a = 24 \Rightarrow \boxed{v_a = 8V}$$

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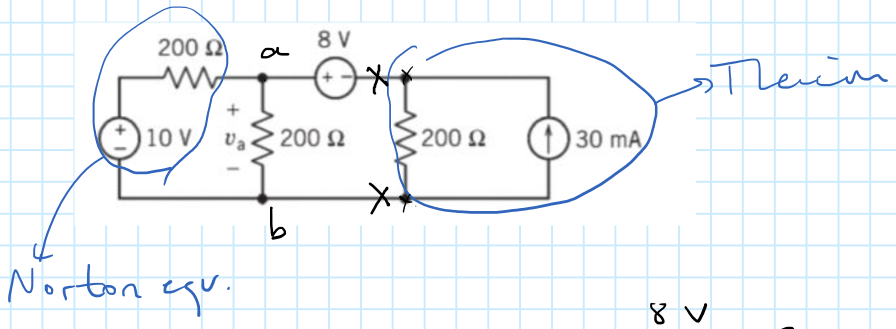
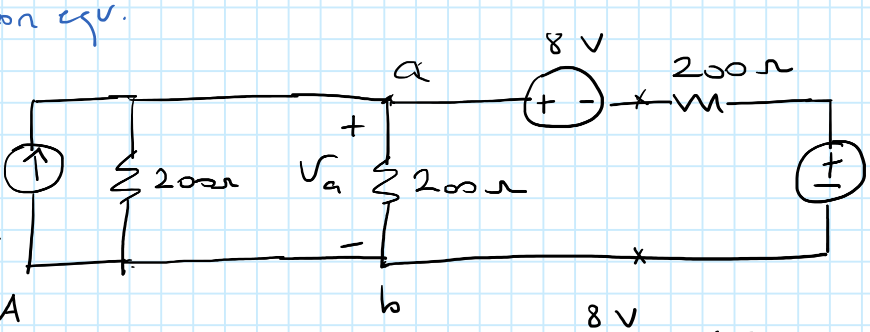
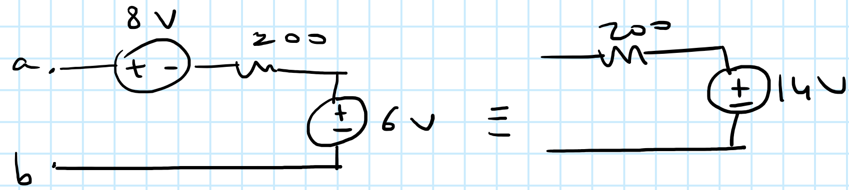
b-) Source transformation = Thevénin → Norton → Thevénin ..

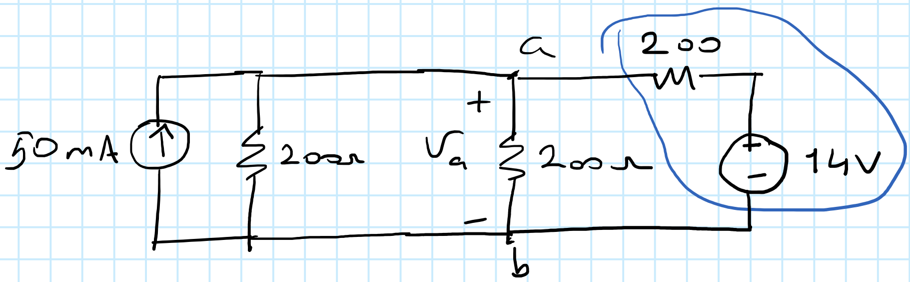


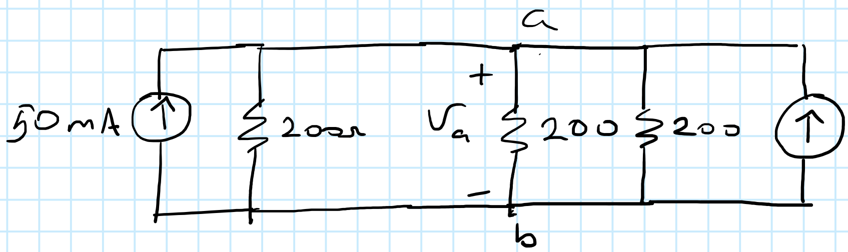


$\frac{10}{200} = 0.05 = 50\,mA$

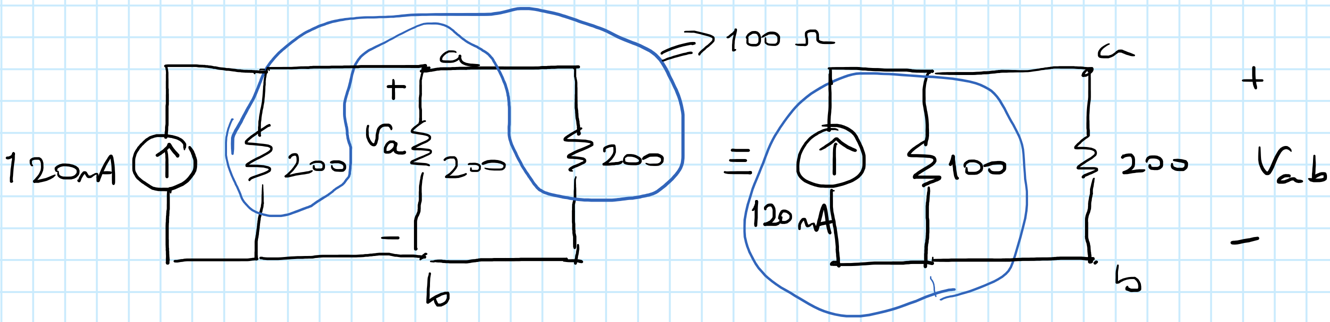
$(30\,mA)(200) = 6\,V.$







$\frac{14}{200} = 70\,mA.$



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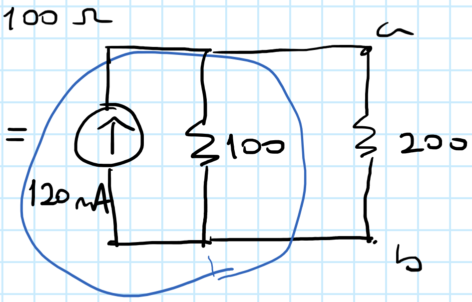
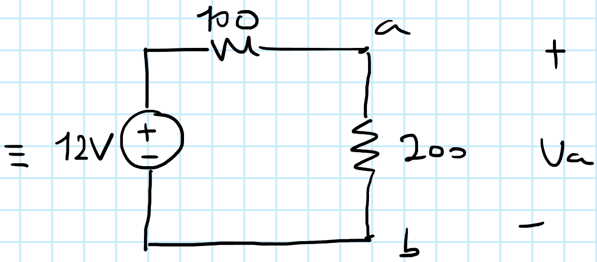




$$V_a = 12 \cdot \frac{200}{200+100} = \cancel{12}^{\,4} \cdot \frac{2\cancel{0}\cancel{0}}{\cancel{3}\cancel{0}\cancel{0}} = 8\,V.$$

4-)

a-) Calculate the value of R for maximum power.
b-) Determine the maximum power absorbed by R.

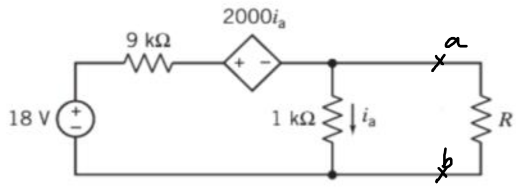
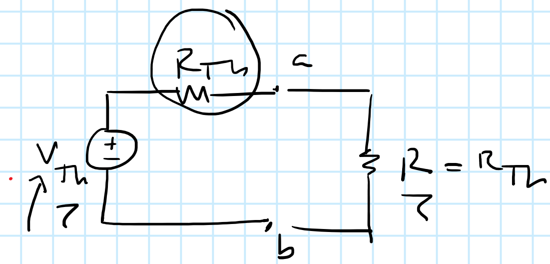
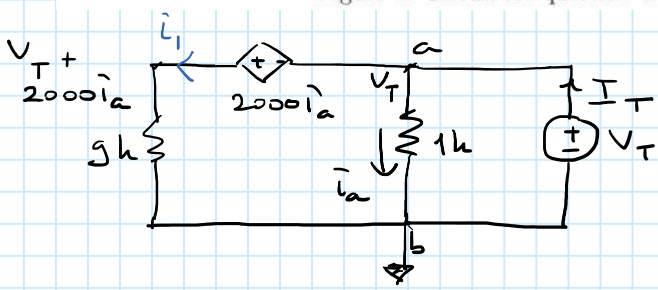


Figure 4: Circuit for question 4.

– Thevenin btw. a and b:





$$R_{Th} = \frac{V_T}{I_T}$$

KCL at node a:

$$i_1 + i_a - I_T = 0 \quad \text{—(1)}$$

$$i_a = \frac{V_T}{1k} \quad \text{—(2)}$$

Substitute (2) in (1)

$$i_1 + \frac{V_T}{1k} - I_T = 0 \quad \text{—(3)}$$

$$\frac{V_T + 2000\, i_a}{9k} + \frac{V_T}{1k} = I_T$$

$$\frac{V_T + \cancel{2000} \cdot \left(\frac{V_T}{\cancel{1k}}\right)}{9k} + \frac{V_T}{1k} = I_T$$

$$\underset{(1)}{\frac{3V_T}{9k}} + \underset{(9)}{\frac{V_T}{1k}} = I_T$$

$$\frac{3V_T + 9V_T}{9k} = I_T$$

$$\frac{12 V_T}{9k} = I_T$$

$$\Rightarrow \frac{V_T}{I_T} = \frac{9000}{12} = 750\,\Omega = R_{Th}.$$

$$\boxed{R = R_{Th} = 750\,\Omega}$$

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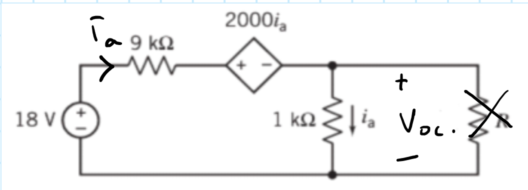
To find V_{Th}: $V_{Th} = V_{oc}$.



KVL for the mesh:

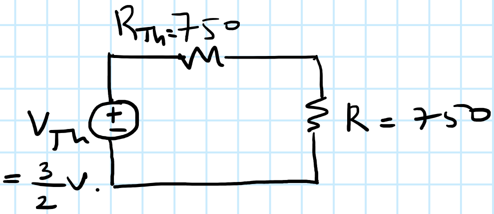
$$-18 + i_a(9k) + 2000\, i_a + i_a(1k) = 0$$

$$12000\, i_a = 18$$

$$i_a = \frac{18}{12000}\ A.$$

$$\Rightarrow V_{Th} = V_{o.c} = i_a \cdot 1k = \frac{18}{12000}(1k) = \frac{\cancel{18}^{\,3}}{\cancel{12}_{\,2}} V = \frac{3}{2} V.$$

b.)



$P_R\big|_{max} = ?$ $(R = 750\,\Omega)$

$$P_R\Big|_{max.} = i^2 \cdot R = \left(\frac{V_{Th}}{2R_{Th}}\right)^2 R_{Th}$$

$$= \frac{V_{Th}^2}{4R_{Th}^{\cancel{2}}} \cdot \cancel{R_{Th}}$$

$$\Rightarrow \boxed{P_R\Big|_{max} = \frac{V_{Th}^2}{4R_{Th}}}$$

$$\Rightarrow P_R\Big|_{max.} = \frac{(3/2)^2}{4 \cdot 750} = \frac{9/4}{4 \cdot 750} = \frac{9}{4} \cdot \frac{1}{4 \cdot 750} = \frac{9}{12000} = \frac{\cancel{9}^{\,3}}{\cancel{12}_{\,4}} mW. = \frac{3}{4} mW.$$

$$= 0.75 mW$$

$$= 750\ \mu W.$$

5-)

a-) Calculate the value of R_L for maximum power.
b-) Determine the maximum power absorbed by R_L.

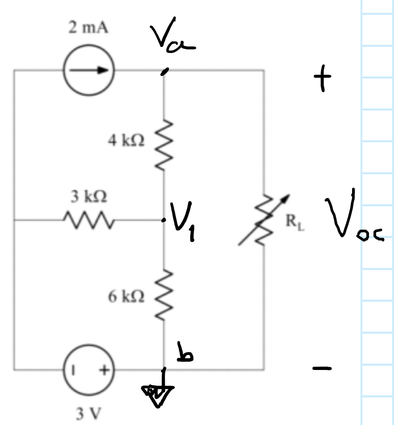


Figure 5: Circuit for question 5.

To find V_{oc}:

Node-voltage method:

KCL at node a:

$$-2mA + \frac{V_a - V_1}{4k} = 0$$

$$\Rightarrow \frac{V_a - V_1}{4\cancel{k}} = 2\cancel{mA}$$

or

$$V_a - V_1 = 8 \quad \text{———} (1)$$

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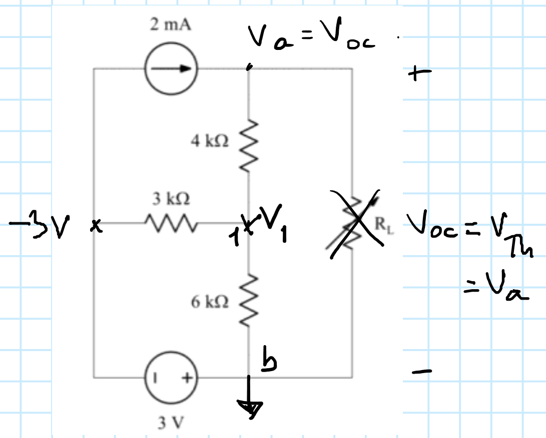


$V_a - V_1 = 8$ ——— (1)

KCL at node 1:

$$\underset{(3)}{\frac{V_1 - V_a}{4k}} + \underset{(4)}{\frac{V_1 - (-3V)}{3k}} + \underset{(2)}{\frac{V_1}{6k}} = 0$$

$$3V_1 - 3V_a + 4V_1 + 12 + 2V_1 = 0$$

$$9V_1 - 3V_a = -12$$

or

$$3V_a - 9V_1 = 12$$

$$V_a - 3V_1 = 4 \quad \text{——— (2)}$$

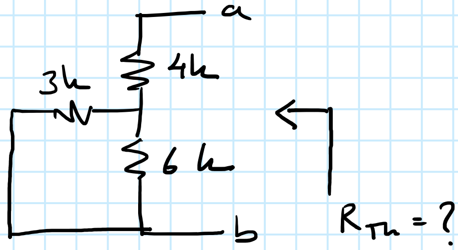
Solve (1) and (2):

$$\begin{aligned} V_a - V_1 &= 8 \\ + \quad -V_a + 3V_1 &= -4 \\ \hline 2V_1 &= 4 \end{aligned}$$

$2V_1 = 4 \Rightarrow V_1 = 2$, $V_a = 10\,V$.

$\Rightarrow V_{Th} = 10\,V$.

R_{Th}:

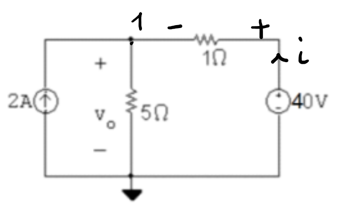


$$\Rightarrow R_{Th} = 4k + \underbrace{(3k \parallel 6k)}_{2k} = 6k\Omega$$

b-)

$$P_R\Big|_{max} = \frac{V_{Th}^2}{4R_{Th}} \Rightarrow P_R = \frac{(10)^2}{4.6000} = \frac{1\cancel{00}}{4.60\cancel{00}} = \frac{1}{240}\,W. \cong 4.2\,mW.$$

6-)



Find V_0, P_{40V} and $P_{5\Omega}$ for the circuit in Fig.1.

Figure 1: Circuit for question 1.

KCL at node 1:

$$-2 + \frac{V_o}{5} + \frac{V_o - 40}{1} = 0$$

$$\frac{V_o}{5} + V_o = 42$$

$$\frac{6V_o}{5} = 42 \Rightarrow V_o = \frac{\overset{7}{\cancel{42}}.5}{\cancel{6}} = 35V$$

$$P_{40V} = -i.V = -i\,(40) \Rightarrow i = \frac{40 - V_o}{1} = 40 - 35 = 5A.$$

$$\Rightarrow P_{40V} = -(5A)(40V) = -200\,W., \quad P_{5\Omega} = \frac{V_o^2}{R} = \frac{35^2}{5} = \frac{\overset{7}{\cancel{35}}.35}{\cancel{5}} = 245\,W.$$

P33a

Monday, November 8, 2021 12:34 PM

Alternatively,

a-) Calculate the value of R_L for maximum power.
b-) Determine the maximum power absorbed by R_L.

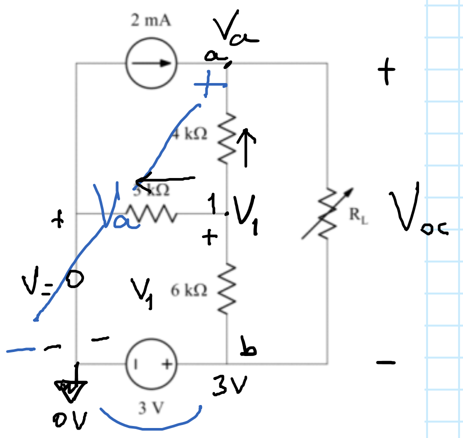


Figure 5: Circuit for question 5.

To find V_{Th}: Use node-voltage method.

KCL at node a:

$$-2mA + \frac{V_a - V_1}{4k} = 0$$

$$V_a - V_1 = 2\cancel{m}A \cdot 4\cancel{k}$$

$$V_a - V_1 = 8 \quad \text{——(1)}$$

KCL at node 1:

$$\frac{V_1 - V_a}{4\cancel{k}} + \frac{V_1}{3\cancel{k}} + \frac{V_1 - 3}{6\cancel{k}} = 0$$

(3) (4) (2)

$$3V_1 - 3V_a + 4V_1 + 2V_1 - 6 = 0$$

$$9V_1 - 3V_a = 6$$

——(2)

Solve (1) and (2) simultaneously,

$$\cancel{V_a} - V_1 = 8$$

$$+\ 3V_1 - \cancel{V_a} = 2$$

$$2V_1 = 10 \Rightarrow V_1 = 5V. \Rightarrow V_a = 13V$$

$$V_{Th} = V_{oc} = V_a - 3V = 13 - 3 = 10V.$$

P34

Saturday, October 31, 2020 11:52 AM

7-)

Find v_1 and v_2 for the circuit in Fig.2.

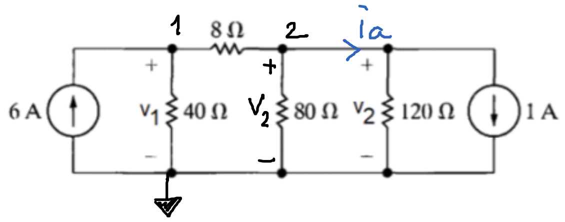


Figure 2: Circuit for question 2.

KCL at node 1:

$$-6+\frac{v_1}{40}+\frac{v_1-v_2}{8}=0 \quad \text{—}(1)$$

KCL at node 2:

$$\frac{v_2-v_1}{8}+\frac{v_2}{80}+i_a=0 \quad \text{—}(2)$$

Also, $i_a = \frac{v_2}{120}+1$ —(3)

Substitute (3) in (2):

$$\frac{v_2-v_1}{8}+\frac{v_2}{80}+\frac{v_2}{120}+1=0 \quad \text{—}(4)$$

Solve (1) and (4) simultaneously:

$$\underset{(1)}{\frac{v_1}{40}}+\underset{(5)}{\frac{v_1-v_2}{8}}=6 \quad \text{—}(1)$$

$$v_1+5v_1-5v_2=6\cdot 40=240 \quad \text{—}(1) \qquad *$$

$$\underset{(30)}{\frac{v_2-v_1}{8}}+\underset{(3)}{\frac{v_2}{80}}+\underset{(2)}{\frac{v_2}{120}}=-1 \quad \text{—}(4)$$

$$30v_2-30v_1+3v_2+2v_2=-240 \quad \text{—}(4)$$

$$\cancel{35}v_2-\cancel{30}v_1=-\cancel{240}$$

$$7v_2-6\cancel{v_1}=-48 \quad \text{—}(4) \qquad *$$

$$+\ 6\cancel{v_1}-5v_2=240 \quad \text{—}(1)$$

$$7v_2-5v_2=240-48$$

$$2v_2=192 \Rightarrow v_2=96\text{ V}.$$

$$6v_1=240+5(96) \Rightarrow v_1=\frac{240+5(96)}{6}=120\text{ V}.$$

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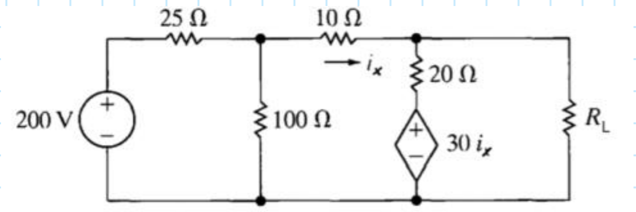
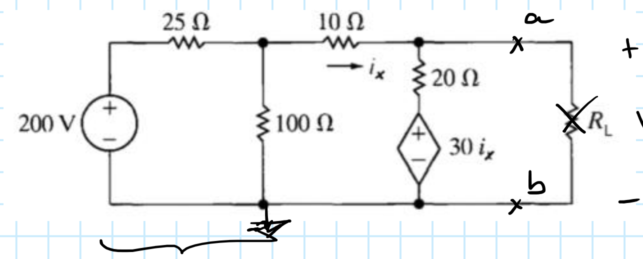
Ex:

For the circuit below,



Find R_L for maximum power transfer. Find P_{R_L} at max. power.

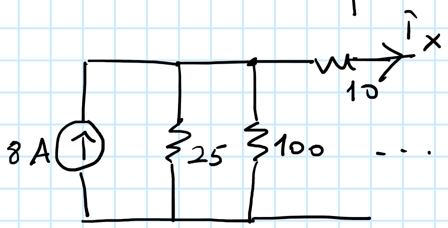
Ans:



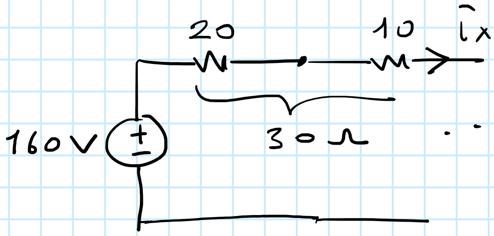
We find the Thevenin circuit w.r.t. a and b.

$V_{o.c} = ?$

Source Transformation

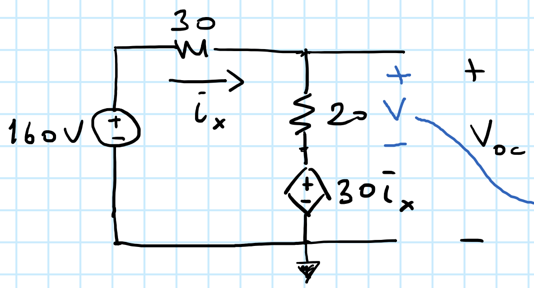


... ⇒



... ⇒

$25 \| 100 = 20\,\Omega$.



Mesh Equation (KVL) for mesh 1:

$-160 + 30 i_x + V_{oc} = 0$ ——(1)

Ohm's law across 20 Ω resistor:

$V_{oc} - 30 i_x = 20 i_x$ ——(2)

$\hookrightarrow i_x = \frac{V_{oc}}{50}$

Substitute i_x into (1),

$$3\cancel{0}\left(\frac{V_{oc}}{5\cancel{0}}\right) + V_{oc} = 160 \Rightarrow \frac{3V_{oc}}{5} + V_{oc} = 160 \Rightarrow \frac{8V_{oc}}{5} = 160$$

$$\Rightarrow V_{oc} = \frac{5 \cdot \cancel{160}^{20}}{\cancel{8}} = 100. = V_{Th}.$$

→ To find R_{Th}, we can use the test voltage, V_T, method:

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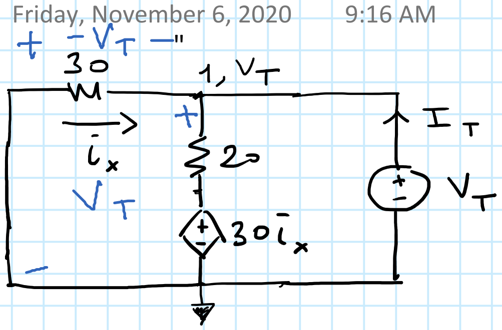


KCL at node 1:

$$-i_x + \frac{V_T - 30 i_x}{20} - I_T = 0 \quad \text{——(1)}$$

Ohm's law across $30\,\Omega$ resistor:

$$30 i_x = -V_T \quad \text{——(2)}$$

$$\Rightarrow i_x = \frac{-V_T}{30}$$

Substitute i_x in (1):

$$\frac{V_T}{30} + \frac{V_T - \cancel{30}\left(\frac{-V_T}{\cancel{30}}\right)}{20} = I_T \Rightarrow \underset{(1)}{\frac{V_T}{30}} + \underset{(3)}{\frac{V_T}{10}} = I_T \Rightarrow$$

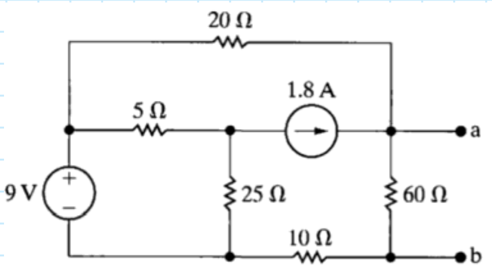
$$\frac{4 V_T}{30} = I_T \qquad \Rightarrow R_{Th} = \frac{V_T}{I_T} = \frac{30}{4} = 7.5\,\Omega.$$

To find P_{RL}:

$$P_{RL} = \frac{V_{Th}^2}{4 R_{Th}} = \frac{(100)^2}{4(7.5)} \simeq 333 \text{ W}.$$

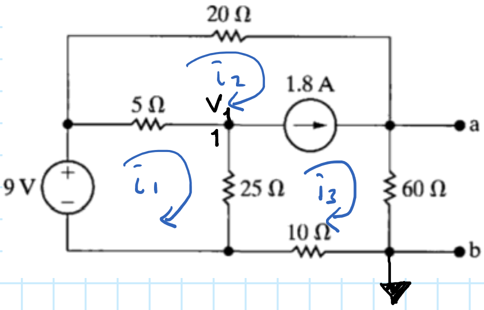
Ex:

For the circuit below, find the Thevenin circuit w.r.t. a-b.



Ans:

To find V_{Th}:



KVL for mesh 1:

$$-9 + 5(i_1 - i_2) + 25(i_1 - i_3) = 0 \quad \text{——(1)}$$

KVL for mesh 2:

$$20 i_2 + (V_{oc} - V_1) + 5(i_2 - i_1) = 0$$

$\Rightarrow$ 5 unknown already $\Rightarrow$ Let us not use "Mesh-current" method.

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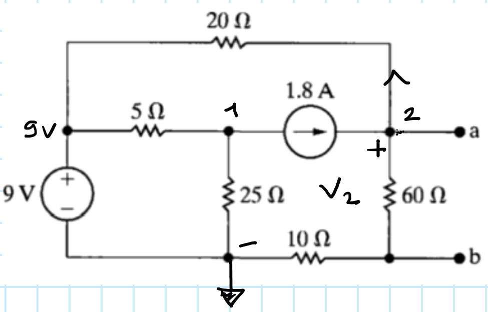


$+$ $V_{o.c} = V_{Th}$ $-$

KCL at node 2:

$$\frac{V_2 - 9}{20} - 1.8 + \frac{V_2}{70} = 0$$

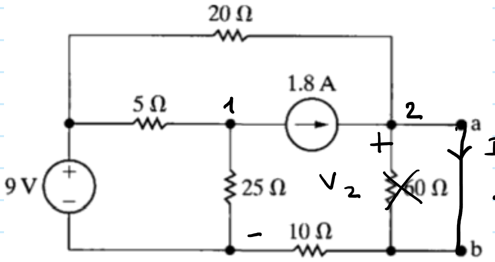
$$\frac{V_2 - 9}{20} + \frac{V_2}{70} = 1.8$$

$\vdots$

$$\Rightarrow V_2 = 35V.$$

$$\Rightarrow V_{Th} = V_{oc} = 35 \cdot \frac{60}{60+10} \text{ (Voltage division)} = 30V.$$

To find R_{Th}:



$I_{s.c} = ?$ s.c.

KCL at node 2:

$$\frac{V_2 - 9}{20} - 1.8 + \frac{V_2}{10} = 0$$

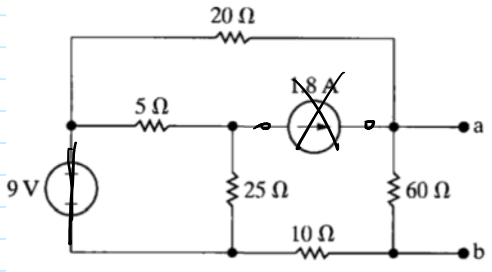
$$\Rightarrow V_2 = 15V.$$

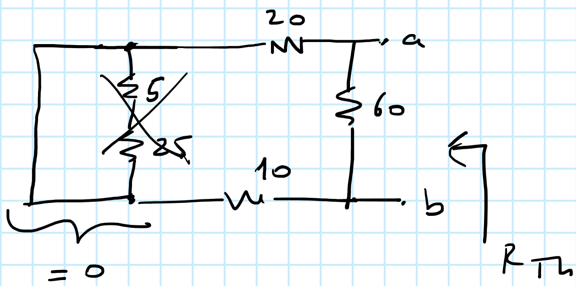
Using Ohm's law across 10Ω:

$$\Rightarrow I_{sc} = \frac{V_2}{10} = 1.5A.$$

$$\Rightarrow R_{Th} = \frac{V_{Th}}{I_{s.c}} = \frac{30}{\frac{3}{2}} = \frac{\cancel{30}^{10} \cdot 2}{\cancel{3}} = 20\ \Omega.$$

Alternatively, we could use the de-activation technique:



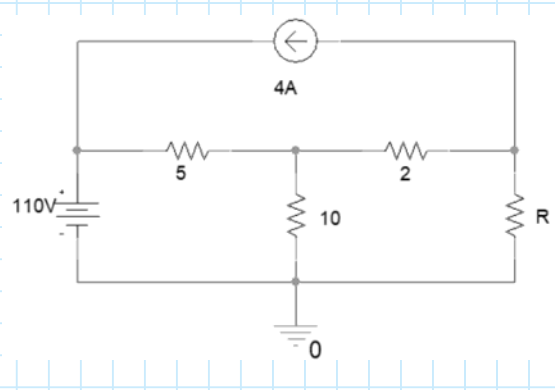


$$\Rightarrow R_{Th} = (20+10) \| 60 = 30 \| 60$$

$$= \frac{\cancel{30} \cdot 60}{\cancel{90}\ 3} = 20\ \Omega. //$$

Ex:

For the circuit given below, find R for max. power transfer and $P_{R_L} = ?$



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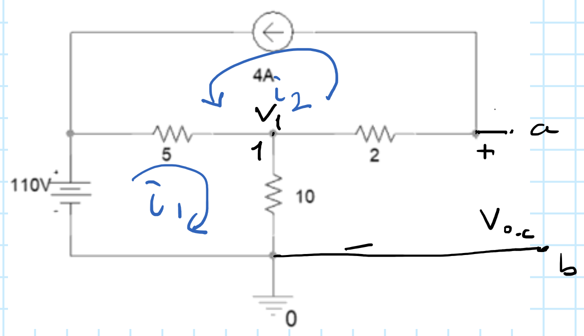
Ans :



KVL in mesh 1 :

$-110 + 5(i_1 + 4) + 10 i_1 = 0$ —— (1)

$15 i_1 = 110 - 20 = 90$

$\Rightarrow i_1 = 6A.$

Ohm's law across 10Ω resistor:

$V_1 = i_1 \cdot 10 = 6 \cdot 10 = 60V$

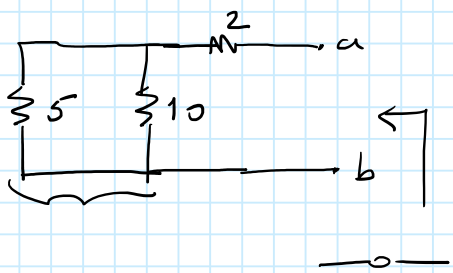
Ohm's law across 2Ω resistor:

$\underset{60}{V_1} - V_{oc} = 2(4) = 8V.$

$\Rightarrow V_{oc} = 60 - 8 = 52V.$

$\Rightarrow V_{Th} = 52V.$

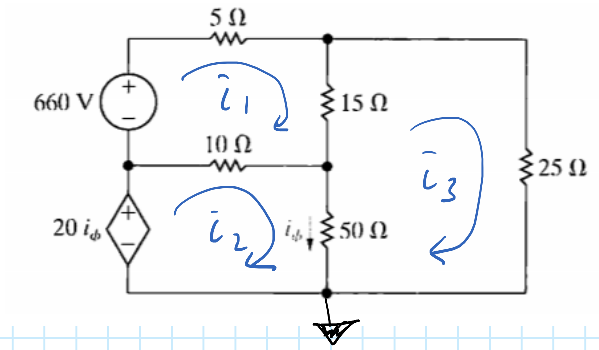
To find R_{Th}:



$R_{Th} = 2 + (5 \| 10) \cong 5.3\Omega.$

Ex:

Given the circuit below, find the power delivered by the dependent source :



Ans:

KVL in mesh 1 :

$-660 + 5 i_1 + 15(i_1 - i_3) + 10(i_1 - i_2) = 0$

$\Rightarrow 30 i_1 - 10 i_2 - 15 i_3 = 660$ —— (1)

KVL in mesh 2:

$-20 i_\phi + 10(i_2 - i_1) + 50(i_2 - i_3) = 0$

Also, $i_\phi = i_2 - i_3$

or $-10 i_1 + 60 i_2 - 50 i_3 = 20 i_\phi = 20 i_2 - 20 i_3$

or $-10 i_1 + 40 i_2 - 30 i_3 = 0$ —— (2)

KVL in mesh 3 :

$50(i_3 - i_2) + 15(i_3 - i_1) + 25 i_3 = 0$

$-15 i_1 - 50 i_2 + 90 i_3 = 0$ —— (3)

Let us solve (1), (2) and (3) simultaneously,

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$$30i_1 - 10i_2 - 15i_3 = 660$$

or

$$6i_1 - 2i_2 - 3i_3 = 132 \quad \text{——(1)}$$

Similarly, eqn (2) also simplifies

$$-10i_1 + 40i_2 - 30i_3 = 0$$

$$i_1 - 4i_2 + 3i_3 = 0 \quad \text{——(2)}$$

Also, eqn (3) can be re-written as

$$-15i_1 - 50i_2 + 90i_3 = 0$$

$$3i_1 + 10i_2 - 18i_3 = 0 \quad \text{——(3)}$$

$$\left.\begin{aligned} 6i_1 - 2i_2 - 3i_3 &= 132 \quad \text{——(1)} \\ i_1 - 4i_2 + 3i_3 &= 0 \quad \text{——(2)} \\ 3i_1 + 10i_2 - 18i_3 &= 0 \quad \text{——(3)} \end{aligned}\right\} \underbrace{\begin{bmatrix} 6 & -2 & -3 \\ 1 & -4 & +3 \\ 3 & 10 & -18 \end{bmatrix}}_{A} \cdot \underbrace{\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix}}_{i} = \underbrace{\begin{bmatrix} 132 \\ 0 \\ 0 \end{bmatrix}}_{b}$$

$$i = A^{-1} \cdot b$$

Solve i_1 from (2)

$$i_1 = 4i_2 - 3i_3 \quad \text{——(4)}$$

Substitute (4) into (3):

$$3(4i_2 - 3i_3) + 10i_2 - 18i_3 = 0$$

or

$$12i_2 - 9i_3 + 10i_2 - 18i_3 = 0$$

$$22i_2 = 27i_3 \Rightarrow i_3 = \frac{22}{27} i_2 \quad \text{——(5)}$$

– You must show your calculations in the exam, You can not use Matlab to find the result.

Substitute (5) in (1) and (2)

$$6i_1 - 2i_2 - \cancel{3}\left(\frac{22}{\cancel{27}_9} i_2\right) = 132 \quad \text{——(1)} \Rightarrow 6\cancel{i_1} - 2i_2 - \frac{22}{9} i_2 = 132 \quad \text{——(6)}$$

$$i_1 - 4i_2 + \cancel{3}\left(\frac{22}{\cancel{27}_9} i_2\right) = 0 \quad \text{——(2)} \Rightarrow (-6)\left(i_1 - 4i_2 + \frac{22}{9} i_2\right) = (0) \cdot (-6)$$

$$= -\cancel{6i_1} + 24i_2 - \frac{6 \cdot 22}{9} i_2 = 0 \quad \text{——(7)}$$

Add (6) and (7)

$$22i_2 - \frac{22}{9} i_2 - \frac{6 \cdot 22}{9} i_2 = 132$$

$$22i_2\left(\underset{(9)}{1} - \underset{(1)}{\frac{1}{9}} - \underset{(3)}{\frac{\cancel{6}^{2}}{\cancel{9}_3}}\right) = 132$$

$$\Rightarrow 22i_2\left(\frac{9 - 1 - 6}{9}\right) = 132 \Rightarrow i_2 = \frac{132 \cdot 9}{22 \cdot 2} = 27\,A. \Rightarrow i_1 = 42A,\ i_3 = 22A.$$

$$\Rightarrow i_\phi = 5A. \quad \Rightarrow 20i_\phi = 100\,V$$

$$P_{20i_\phi} = -i \cdot v = -100\, i_2 = -100 \cdot 27 = -2.7\,kW.$$

or

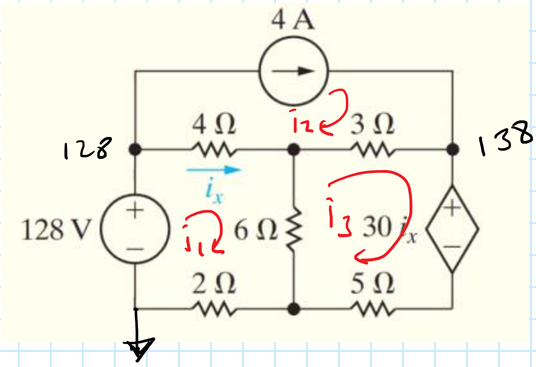
$$P_{20i_\phi} = 2.7\,kW \text{ delivered power.}$$

(– power)

P40

Tuesday, November 10, 2020 2:49 PM

Ex:



KVL for mesh 1:

$-128 + 4 i_x + 6(i_1 - i_3) + 2 i_1 = 0$ —(1)

KVL for mesh 2:

$(128 - v_1) + 3(i_2 - i_3) + 4(i_2 - i_1) = 0$ —(2)

KVL for mesh 3:

$6(i_3 - i_1) + 3(i_3 - i_2) + 30 i_x + 5 i_3 = 0$ —(3)

$i_2 = 4$

$i_x = i_1 - 4$

$4(i_1 - 4) + 6 i_1 - 6 i_3 + 2 i_1 = 128$

$12 i_1 - 6 i_3 = 144$ —(1)

$128 - v_1 + 12 - 3 i_3 + 16 - 4 i_1 = 0$

$-4 i_1 - 3 i_3 - v_1 = -156$ —(2)

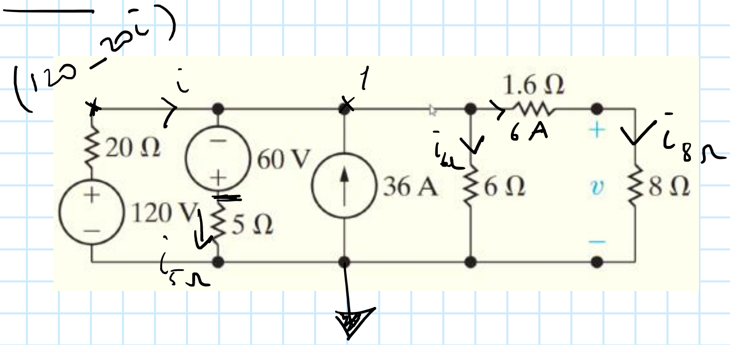
$6 i_3 - 6 i_1 + 3 i_3 - 12 + 30(i_1 - 4) + 5 i_3 = 0$

$24 i_1 + 14 i_3 = 132$ —(3)

$$\begin{bmatrix} 12 & -6 & 0 \\ +4 & +3 & +1 \\ 24 & 14 & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_3 \\ v_1 \end{bmatrix} = \begin{bmatrix} 144 \\ 156 \\ 132 \end{bmatrix}$$

$i_1 = 9, \; i_2 = -6, \; V_1 = 138$

Ex!



$v = 48\,V.$

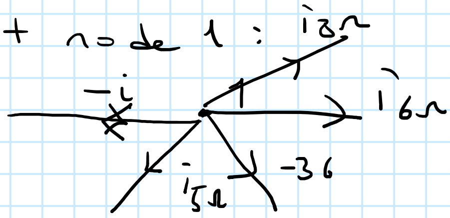
$P_{120} = 374.4W.$

$i_{8\Omega} = \frac{v}{8} = \frac{48}{8} = 6A.$

$i_{5\Omega} = \frac{180 - 20i}{5} = 36 - 4i$

$i_{6\Omega} = \frac{120 - 20i}{6} = 20 - \frac{10}{3} i$

KCL at node 1:



$-i + i_{5\Omega} + i_{6\Omega} + i_{8\Omega} - 36 = 0$

$-i + 36 - 4i + 20 - \frac{10}{3} i + 6 - 36 = 0$

$-5i - \frac{10}{3} i = -26$

$i(5 + \frac{10}{3}) = 26$

$i = \frac{26}{\frac{25}{3}} = \frac{26 \cdot 3}{25}$ A.

$P_{120V} = 120 \cdot \frac{26 \cdot 3}{25} = 374.4W.$

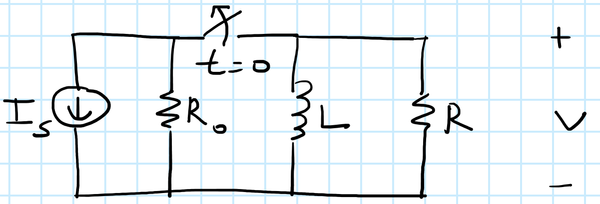
Final Exam Starts from Here

P41 - RL & RC Circuits -

Tuesday, November 10, 2020 4:35 PM

Natural Response of an RL-Circuit: (Discharging the current of the inductor)

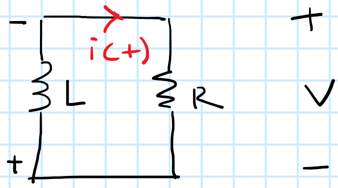
Consider the following circuit:



Norton circuit is used to charge L for t<0

RL Circuit

⇒ For $t \geq 0$



Assume that $L\frac{di}{dt} = 0$, $t < 0$ (the inductor is a short circuit before the release of the switch.)

⇒ For t<0, currents in R_0 and R = 0.

All current flows through the L.

For t=0, the inductor begins to release its energy.

For $t \geq 0$, the KVL equation for i(t) is:

$$\boxed{L\frac{di(t)}{dt} + R\,i(t) = 0}$$

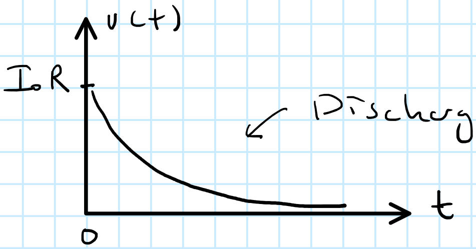
(1st order ODE)
↳ ordinary Diff. Equ.

$i(0^-) = i(0^+) = I_0$
↳ just after 0.
↳ just before 0

⇒ $i(0) = I_0 = I_s$

⇒ The solution is: $\boxed{i(t) = I_0 \cdot e^{-\left(\frac{R}{L}\right)t}}$ (A), and

$v(t) = i(t)\,R$ gives $\boxed{v(t) = I_0 R\, e^{-\left(\frac{R}{L}\right)t}}$ (V), $t \geq 0$.



Discharging of L (natural response)

The power dissipation: $p = v \cdot i = i^2 \cdot R = \frac{v^2}{R}$. Thus,

$$\boxed{p = i(t)^2 \cdot R = I_0^2 R\, e^{-2(R/L)t}}, \quad t \geq 0.$$

The energy delivered to the resistor at any time t:

$$W = \int_0^t p \cdot dx = \int_0^t I_0^2 R \cdot e^{-2\left(\frac{R}{L}\right)x}\, dx = \frac{1}{2} L I_0^2 \left[1 - e^{-2\left(\frac{R}{L}\right)t}\right], \quad t \geq 0$$

Now,

$$\lim_{t \to \infty} W = \lim_{t \to \infty} \left\{ \frac{1}{2} L I_0^2 \left[1 - \underbrace{e^{-2\left(\frac{R}{L}\right)t}}_{0}\right] \right\} = \frac{1}{2} L I_0^2 \text{ (J)}$$ is the initial energy stored in the inductor.

Here, the constant $\left(\frac{L}{R}\right)$ is called the "time constant":

$$\text{Time constant} = \boxed{\tau = \frac{L}{R}}$$

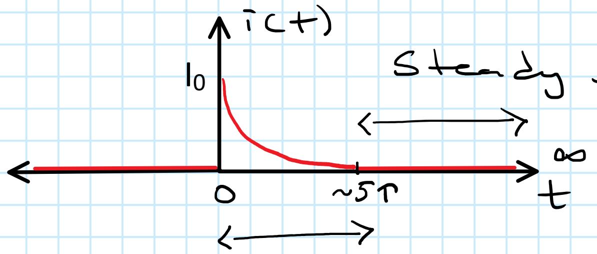
At one τ after $t = 0$,

$$i(\tau) = I_0 \cdot e^{-\left(\frac{R}{L}\right)\left(\frac{L}{R}\right)} = I_0 \underbrace{e^{-1}}_{0.37}$$ $\Rightarrow$ The current is reduced to 37% of its value at $t = 0$.

At 2τ, $i(2\tau) = 0.13\, I_0$ $\Rightarrow$ Reduced to 13%.

At 5τ, it is conventional to assume that $i(t)$ is reduced to zero, and all inductor energy is released.

Remark: Consider the overall response.



Steady State Response: The response that exists a long time after switching

Transient Response: (The response for momentary events)

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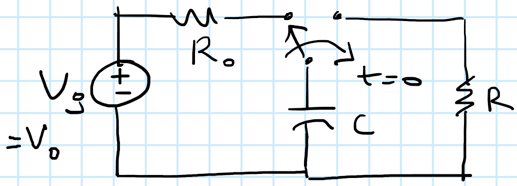
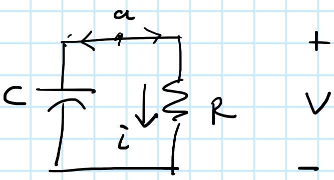
Natural Response of an RC-Circuit:

Consider the following circuit:



For $t \geq 0$, the response is the "natural response":



KCL equation is:

$$C\frac{dV}{dt} + \frac{V}{R} = 0$$

$V(0) = V_0$ (initial capacitor voltage)

The solution is:

$$\boxed{V(t) = V_0 e^{-t/\tau}}, \quad t \geq 0 \quad \text{where } \boxed{\tau = RC} \text{ (time constant)}$$

and

$$i(t) = \frac{V(t)}{R} = \frac{V_0}{R} e^{-t/\tau}, \quad t \geq 0.$$

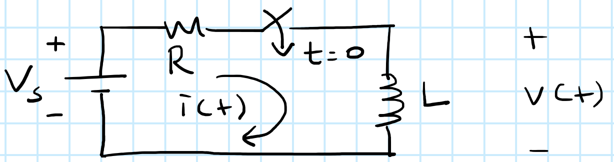
$$P = \frac{V_0^2}{R} e^{-2t/\tau}, \quad t \geq 0.$$

$$W(t) = \frac{1}{2} C\left[V_0^2 \left(1 - e^{-2t/\tau}\right)\right], \quad t \geq 0$$

$$\lim_{t \to \infty} W(t) = \frac{1}{2} C V_0^2 \quad \text{(initial energy stored in the capacitor)}$$

Step Response of an RL-Circuit: (Charging the inductor)

Consider the following circuit:



After the switch is closed, the KVL eqn is:

$$V_s = R\,i(t) + L\frac{di(t)}{dt}$$

or

$$\frac{di(t)}{dt} = \frac{1}{L}\left[V_s - R\,i(t)\right]$$

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$$\frac{di(t)}{dt} = \frac{1}{L}\left[V_s - R\,i(t)\right]$$

or

$$\frac{di(t)}{dt} + \frac{R}{L}\,i(t) = \frac{V_s}{L}$$

(1^{st} order ODE, linear nonhomogenous)

The initial condition is $i(0) = I_0 = 0$

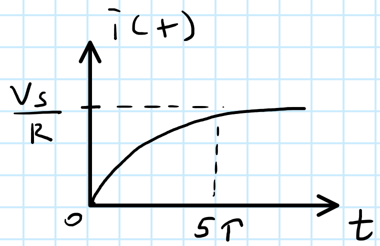
The solution is :

$$\boxed{i(t) = \frac{V_s}{R} + \left(I_0 - \frac{V_s}{R}\right) e^{-\left(\frac{R}{L}\right)t}, \quad t \geq 0}$$

For $I_0 = 0$:

(Step response of 1^{st} order RL circuit)

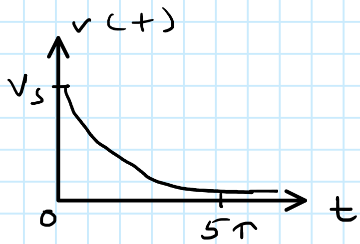
$$\boxed{i(t) = \frac{V_s}{R} - \frac{V_s}{R}\, e^{-\left(\frac{R}{L}\right)t}}$$



The voltage can be obtained from:

$$V = L\frac{di(t)}{dt} = \cancel{L}\left(-\frac{R}{\cancel{L}}\right)\left(\underbrace{I_0}_{0} - \frac{V_s}{R}\right) e^{-\left(\frac{R}{L}\right)t}$$

$$\boxed{v(t) = V_s\, e^{-\left(\frac{R}{L}\right)t} \quad (V), \; t \geq 0.}$$



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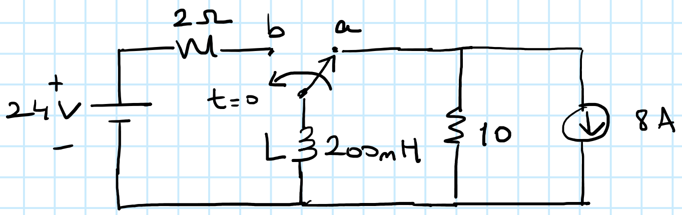
Ex:

For the following circuit

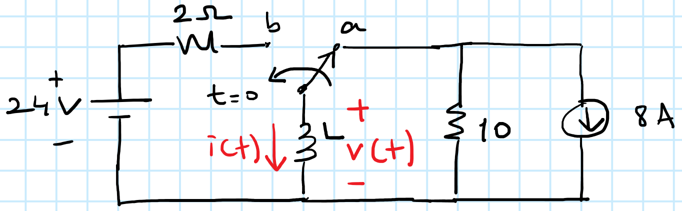


For $t<0$, the circuit is already settled.

a-) Find $i(t)$, $t \geq 0$

b-) What is the initial voltage across the inductor just after the switch is moved to position b.

Ans:



For $t<0$, $i(0) = I_o = -8\,A$.

$i(\infty) = I_{final} = \dfrac{24\,V}{2\,\Omega} = 12\,A$.

and $\tau = \dfrac{L}{R} = \dfrac{200\,mH}{2} = 100\,ms$.

a-) Using the step response for RL circuit:

$$i(t) = \underbrace{\frac{V_s}{R}}_{I_{final}} + (\underbrace{I_o}_{-8\,A.} - \underbrace{\frac{V_s}{R}}_{I_{final}.})e^{-\frac{t}{\tau}}, \quad V_s = 24,\ R = 2\,\Omega.$$

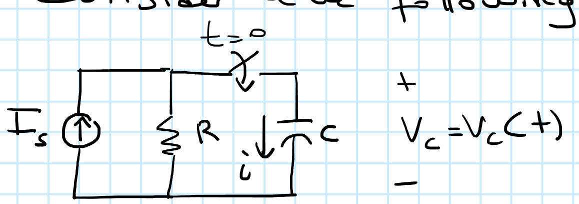
$$\Rightarrow i(t) = 12 - 20e^{-10t}\ A.$$

b-) $v = L \cdot \dfrac{di(t)}{dt} = 0.2\left(200e^{-10t}\right) = 40e^{-10t}\ (V),\ t \geq 0.$

$$\Rightarrow v(0^+) = 40\ V. //$$

Step Response of an RC-Circuit:

Consider the following circuit:



The KCL eqn is: for $t \geq 0$

$$C\frac{dV_c(t)}{dt} + \frac{V_c(t)}{R} = I_s$$

or

$$\frac{dV_c(t)}{dt} + \frac{1}{RC}V_c(t) = \frac{I_s}{C} \quad (1^{st}\ \text{order ODE})$$

The solution is:

$$V_c(t) = I_sR + [V_o - I_sR]e^{-t/RC}, \quad t \geq 0.$$

and

$$i(t) = \left[I_s - \frac{V_o}{R}\right]e^{-t/RC}, \quad t \geq 0.$$

$5\tau = 10$

$RC = 2$

$R = 2$

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General Response for Step & Natural Responses:

of 1st order diff. equation.

The general response is given as:

$$\boxed{x(t) = X_f + [x(t_0) - X_f]e^{-(t-t_0)/\tau}}$$

In another interpretation;

The Unknown variable = The final value of the variable +

$$\left[\text{The initial value of the variable} - \text{The final value of the variable}\right] e^{\frac{-(t-t_0)}{\tau}}$$

When computing the step and/or natural responses of an RL or RC circuits using this general formulation, the following steps are used:

1-) Identify the variable of interest:

For an RL circuit → Inductor current.

For an RC circuit → Capacitor voltage.

2-) Determine the initial value of the variable, which is its value at t_0. If chosen as capacitor voltage or inductor current as the variable of interest, then

$$V_c(t_0^-) = V_c(t_0^+) \quad \text{and} \quad I_L(t_0^-) = I_L(t_0^+)$$

Otherwise, we must use the t_0^+ value of the variable as initial value.

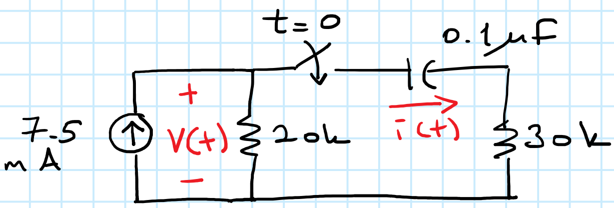
3-) Calculate the final value of the variable at $t \to \infty$.

4-) Calculate the time constant.

5-) Substitute all variables into the general formulation.

Ex:

For the following circuit



For $t<0$, the current is already settled. Initial charge of the capacitor is 0.

Find, a-) $i(t)$ for $t \geq 0$

b-) $v(t)$, $t \geq 0$.

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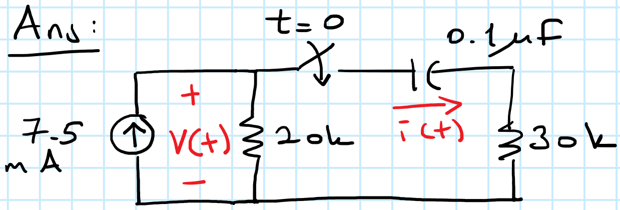
Ans:



a-)

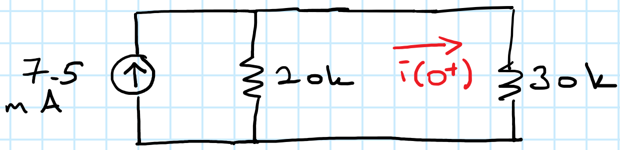
Steps: 1-) Variable of interest is $i(t)$.

2-) Variable of interest is the cap. current.

$t_0 = 0$

$i(0^+) = ?$

At $t = 0^+$ → At this instant, the rate of change in current is max.



For the capacitor, $i(t) = C \frac{dv_c(t)}{dt}$ max. max.

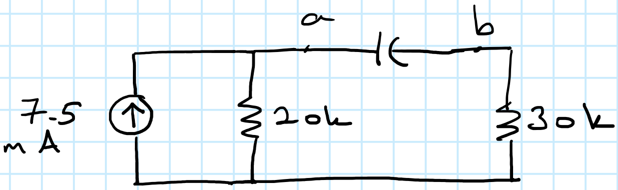
When is $i(t)$ max? ⇒ short circuit.

$i(0^+) = (7.5mA) \cdot \frac{20\cancel{k}}{50\cancel{k}} = 3\,mA$ (from current division)

Note that $i(0^-) = 0$, $i(0^+) = 3\,mA$ $\Big\}$ $i(0^-) \neq i(0^+)$

3-) $i(t = \infty) = i_{final} = 0$ and

4-) $\tau = RC = ?$



$R = R_{Th}$ w.r.t. point a-b.

To find R_{Th}: Deactivate the indep. sources:

⇒ $R_{Th} = 20k + 30k = 50\,k\Omega$.

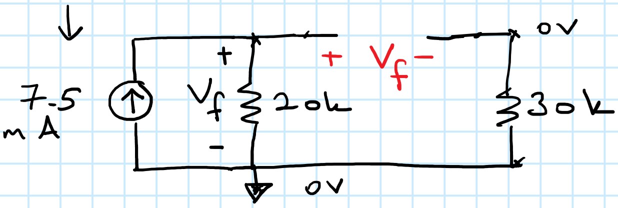
⇒ $\tau = RC = (50k)(0.1\mu F) = 5\,ms$.

5-) $i(t) = 0 + (3 - 0)e^{-t/5\times10^{-3}}$ (mA), $t \geq 0$.

or $i(t) = 3e^{-200t}$, (mA), $t \geq 0$.

b-) Choose the unknown variable as the cap. voltage: Then find $v(t)$.

⇒ $V_c(0^+) = 0V$, $V_f = (7.5)(20) = 150V$, $\tau = 5\,ms$.



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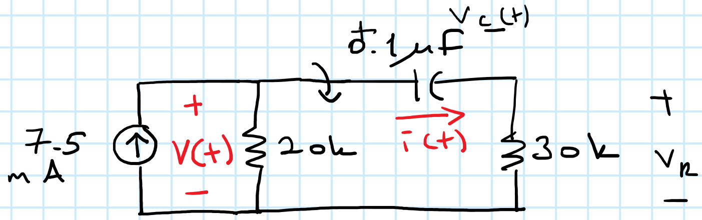
$$\Rightarrow v_c(t) = 150 - 150e^{-200t}, \quad t \geq 0$$



$$\Rightarrow v(t) = v_C(t) + v_R(t), \quad t \geq 0$$

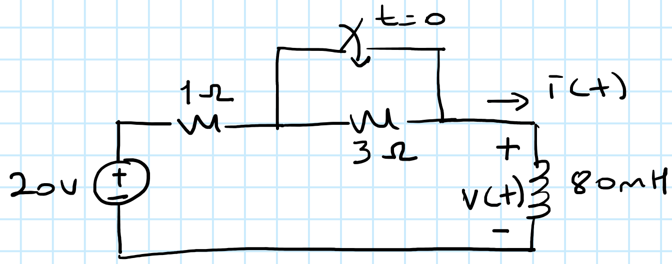
✓ $\quad i(t) \cdot R$

$$\Rightarrow v(t) = 150 - 150e^{-200t} + (30)(3)e^{-200t}$$

$$\Rightarrow v(t) = 150 - 60e^{-200t} \text{ (V)}, \quad t \geq 0. \;//$$

Ex:

For the circuit



Find v(t) and i(t) for $t \geq 0$.

Ans:

Variable of interest → inductor current i(t).

$$I_0 = \frac{20V}{4\Omega} = 5A, \quad I_f = \frac{20V}{1\Omega} = 20 A.$$

$$\tau = \frac{L}{R} = \frac{80mH}{R} = \frac{80mH}{1\Omega} = 80ms.$$

$$\Rightarrow i(t) = 20 + (5-20)e^{-t/80ms} = 20 - 15e^{-12.5t} \text{ (A)}, \quad t \geq 0.$$

$$\Rightarrow v(t) = L\frac{di(t)}{dt} = 15e^{-12.5t} \text{ (V)}, \quad t \geq 0.$$

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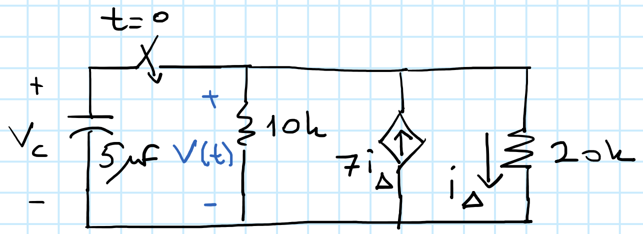
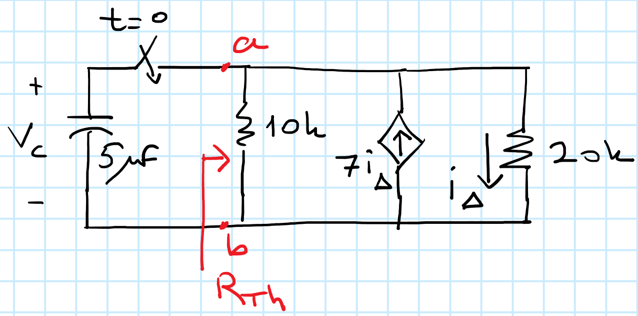
Ex:

For the given circuit, find $V(t)$. ($V_c = 10V$ for $t<0$)

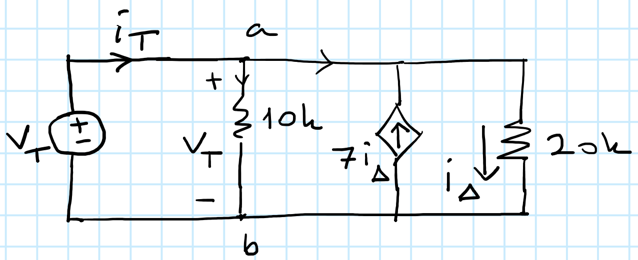


Ans:

- Variable of interest is the cap. voltage ($V_c = V$, $t \geq 0$)
- To determine $V_o(o)$, we can first replace the capacitor terminals by its Thevénin resistance.



To find R_{Th}:



KCL at node a:

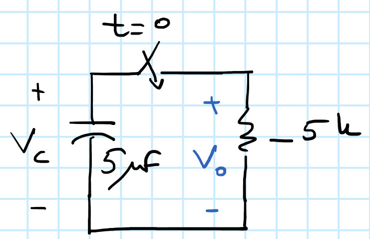
$$\Rightarrow i_T = \left[\frac{V_T}{10k} - 7 i_\Delta + i_\Delta \right]$$

where $i_\Delta = \frac{V_T}{20k}$ (ohm's law across 20kΩ)

$$\Rightarrow i_T = \left[\frac{V_T}{10k} - 7\frac{V_T}{20k} + \frac{V_T}{20k} \right]$$

$$\Rightarrow R_{Th} = \frac{V_T}{i_T} = -5k\Omega .$$

Negative resistance indicates that signal grows.



- $V_c(0^-) = V_c(0^+) = 10V.$
- $V_c(\infty) = V_o(\infty) = 0V.$
- $\tau = RC = (5\mu F)(-5k) = -25ms.$
- $V(t) = \underbrace{V_f}_{0} + \left[\underbrace{V(0)}_{10V.} - \underbrace{V_f}_{0} \right] e^{-[t/(-25ms)]}$

$\Rightarrow V(t) = 10 e^{40t}$, $t \geq 0$ (Unbounded response)

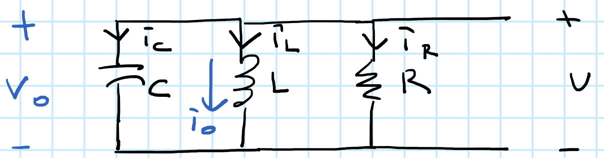
(not realistic!)

RLC Circuits (Chapter 8)

Natural & Step Response:

1-) Natural Response of Parallel RLC Circuit:

The RLC circuit is given as: (parallel)



KCL at the top node:

$$\underbrace{\frac{v}{R}}_{i_R} + \underbrace{\frac{1}{L}\int_0^t v\,d\tau + I_0}_{i_L} + \underbrace{C\frac{dv}{dt}}_{i_C} = 0$$

Differentiate both sides wrt. t:

$$\frac{1}{R}\frac{dv}{dt} + \frac{v}{L} + C\frac{d^2v}{dt^2} = 0$$

Re-arranging this equation,

$$\boxed{\frac{d^2v}{dt^2} + \frac{1}{RC}\frac{dv}{dt} + \frac{v}{LC} = 0}$$ (2^{nd} order differential equation)

The classical approach to solve this equation is to assume a solution in the form which is exponential.

Let $v = Ae^{st}$ be the solution. Then, substitute this solution into the equation:

$$As^2e^{st} + \frac{1}{RC}Ase^{st} + \frac{Ae^{st}}{LC} = 0$$

or

$$Ae^{st}\left(s^2 + \frac{s}{RC} + \frac{1}{LC}\right) = 0$$

A=0 is a trivial solution, It cannot be accepted.

$$\Rightarrow\ s^2 + \frac{s}{RC} + \frac{1}{LC} = 0$$ This equation is called the "characteristic equation".

There are 2 roots:

$$s_{1,2} = \frac{-1}{2RC} \pm \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}}$$

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Thus, $v = A_1 e^{s_1 t}$ and $v = A_2 e^{s_2 t}$ are the 2 solutions.

Also, $\boxed{v = A_1 e^{s_1 t} + A_2 e^{s_2 t}}$ is a solution.

Consider the roots of the characteristic equation in another notation:

$$\boxed{s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}}$$

where

$\boxed{\alpha = \frac{1}{2RC}}$ is called the "Neper frequency".

and

$\boxed{\omega_0 = \frac{1}{\sqrt{LC}}}$ is called the "Resonant radian frequency".

There are 3 possible outcomes:

1-) If $\boxed{\omega_0^2 < \alpha^2}$ $\rightarrow$ Both roots are real. The response is called "overdamped".

2-) If $\boxed{\omega_0^2 > \alpha^2}$ $\rightarrow$ Both roots are complex, and conjugate of each other. That is; $s_1 = a_1 + jb_1$, then $s_2 = a_1 - jb_1$ or visa versa. The response is called "underdamped".

3-) If $\boxed{\omega_0^2 = \alpha^2}$ $\rightarrow$ $s_{1,2}$ are real and equal. The response is called "critically damped".

Ex:

For an RLC circuit, with $R = 200\,\Omega$, $L = 50\,mH$, $C = 0.2\,\mu F$

a) Find the roots of the characteristic equation.

b-) Determine the type of response?

c-) Repeat part a and b for $R = 312.5\,\Omega$.

d-). What values of R causes the response to be critically damped?

Ans:

$$\alpha = \frac{1}{2RC} = \frac{10^6}{2(200)(0.2)} = 1.25 \times 10^4 \text{ rad/s}, \quad \alpha^2 = 1.5625 \times 10^8$$

$$\omega_0^2 = \frac{1}{LC} = \frac{10^3 \cdot 10^6}{(50)(0.2)} = 10^8 \text{ rad}^2/s^2$$

a-) $s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} = -1.25 \times 10^4 \pm \sqrt{1.5625 \times 10^8 - 10^8}$

$\Rightarrow s_1 = -5000$ rad/s, $s_2 = -20000$ rad/s.

b-) Since $\omega_0^2 < \alpha^2$, it is overdamped.

c-) For $R = 312.5\,\Omega$, $\alpha = 8000\ \frac{rad}{s}$, $\alpha^2 = 0.64 \times 10^8\ rad^2/s^2$

and $\omega_0^2 = 10^8 (rad/s)^2$

$$s_1 = -8000 + j6000\ (rad/s), \quad s_2 = -8000 - j6000\ (\frac{rad}{s})$$

The response is underdamped since $\omega_0^2 > \alpha^2$.

d-) For critically damped response, $\alpha^2 = \omega_0^2$

$$\Rightarrow \left(\frac{1}{2RC}\right)^2 = \frac{1}{LC} = 10^8 \Rightarrow \frac{1}{2\underbrace{RC}_{0.2\mu F}} = 10^4 \Rightarrow \boxed{R = 250\,\Omega}$$

The Overdamped Voltage Response:

The solution is in the form:

$\boxed{v(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}}$ where s_1 and s_2 are the roots of the characteristic equation. A_1 and A_2 are the constants to be determined by the initial condition

$\Rightarrow$ Initial conditions: $v(0^+)$ or $\frac{dv(0^+)}{dt}$

Then, $\boxed{v(0^+) = A_1 + A_2}$ and $\boxed{\frac{dv(0^+)}{dt} = s_1 A_1 + s_2 A_2}$.

– The $v(0^+)$ is the initial voltage on the capacitor.

– The value of $\frac{dv(0^+)}{dt}$ can be obtained by finding the current through the capacitor

$$\boxed{\frac{dv(0^+)}{dt} = \frac{i_c(0^+)}{C}}$$

Also,
$$i_C(0^+) = \frac{-V_0}{R} - I_0 .$$

Summary for Finding the "Overdamped" Response:

1-) Find s_1 and s_2

2-) Find $v(0^+)$ and $\frac{dv(0^+)}{dt}$

3-) Find A_1 and A_2 from $v(0^+) = A_1 + A_2$,

$$\frac{dv(0^+)}{dt} = s_1 A_1 + s_2 A_2 .$$

4-) Substitute the values into the solution

$$v(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

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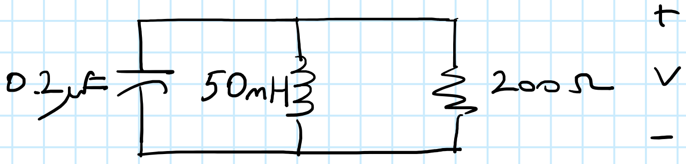
Ex:

For the circuit below, given that $v(0^+) = 12V$, $i_L(0^+) = 30mA$

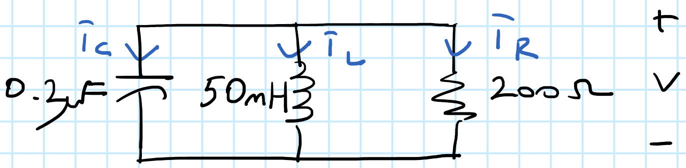


a-) Find the initial current in each branch of the circuit.

b-) Find the initial value of $\frac{dv}{dt}$.

c-) Find the response for $v(t)$.

Ans:



a-) $i_L(0^+) = i_L(0^-) = i_L(0) = 30mA$.

$$i_R(0^+) = \frac{12V}{200} = 60mA.$$

Then, from the KCL at the top node:

$i_C(0^+) = -i_L(0^+) - i_R(0^+) = -90mA$. (the real direction is opposite to our assumption.)

b-) $\frac{dv(0^+)}{dt} = ?$, $i_C(0^+) = C\frac{dv(0^+)}{dt}$

$$\Rightarrow \frac{dv(0^+)}{dt} = \frac{1}{C} i_C(0^+) = \frac{-90 \times 10^{-3}}{0.2 \times 10^{-6}} = -450 \frac{kV}{s}.$$

c-) The roots of the characteristic equation are

$$s_1 = -1.25 \times 10^4 + \sqrt{1.5625 \times 10^8 - 10^8} = -5000 \text{ rad/s}.$$

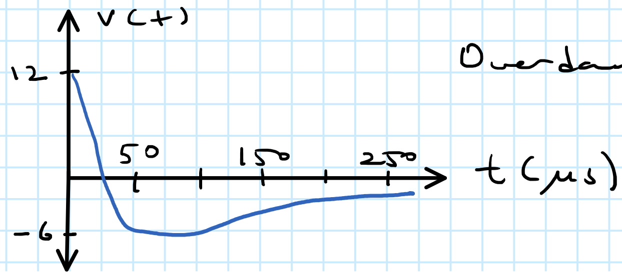
$$s_2 = -1.25 \times 10^4 - \sqrt{1.5625 \times 10^8 - 10^8} = -20000 \text{ rad/s}.$$

To find the coefficients A_1 and A_2:

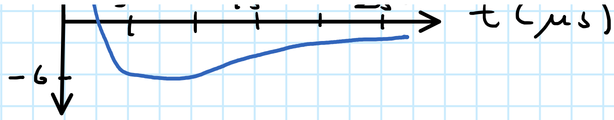
$\boxed{12 = A_1 + A_2}$, $\boxed{-450 \times 10^3 = -5000 A_1 - 20000 A_2}$

$\Rightarrow A_1 = -14V$, $A_2 = 26V$

$$\Rightarrow v(t) = (-14e^{-5000t} + 26e^{-20000t}) \text{ (V)}, \quad t \geq 0.$$



Overdamped!


-6
t (μs)

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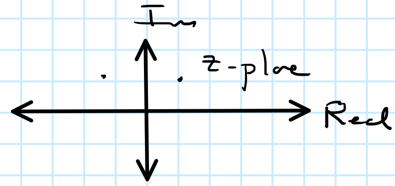


Underdamped Voltage Response:

When $\omega_0^2 > \alpha^2$, the roots are

$$S_1 = -\alpha + \sqrt{-(\omega_0^2 - \alpha^2)} = -\alpha + j\sqrt{\omega_0^2 - \alpha^2} \quad , \; j = i = \sqrt{-1}$$

$$\boxed{S_1 = -\alpha + j\omega_d} \quad , \quad \boxed{S_2 = -\alpha - j\omega_d}$$

where

$\boxed{\omega_d = \sqrt{\omega_0^2 - \alpha^2}}$ is called the "damped radian frequency."

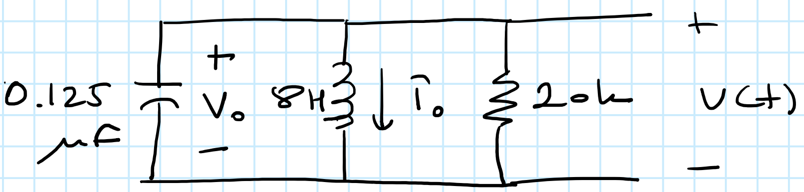
The response is:

$$\boxed{v(t) = B_1 e^{-\alpha t} \cos(\omega_d t) + B_2 e^{-\alpha t} \sin(\omega_d t)}$$

where the coefficients B_1 and B_2 can be obtained from:

$$\boxed{v(0^+) = V_0 = B_1} \quad , \quad \boxed{\frac{dv(0^+)}{dt} = \frac{i_C(0^+)}{C} = -\alpha B_1 + \omega_d B_2 \quad (\alpha: \text{damping factor})}$$

Ex: For the given circuit



Given:

$V_0 = 0\,V$, $I_0 = -12.25\,mA$.

a-) s_1 , s_2 = ?

b-) $v\big|_{t=0} = v(0)$, $\left.\frac{dv}{dt}\right|_{t=0} = ?$

c-) $v(t) = ?$, $t \geq 0$.

Ans:

$$\alpha = \frac{1}{2RC} = 200\ \text{rad/s}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = 10^3\ \text{rad/s}.$$

$\Rightarrow \omega_0^2 > \alpha^2 \Rightarrow$ Underdamped!

Now,

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2} = 100\sqrt{96} = 979.80\ \text{rad/s}.$$

$$S_1 = -\alpha + j\omega_d = -200 + j\,979.8\ \text{rad/s}.$$

$$S_2 = -\alpha - j\omega_d = -200 - j\,979.8\ \text{rad/s}.$$

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b-) $v(0^+) = ?$, $\frac{dv(0^+)}{dt} = ?$

$v(0) = v(0^+) = V_0 = 0\,V.$

To find $\frac{dv(0^+)}{dt}$, we need $i_C(0^+)$.

$\Rightarrow i_R(0^+) = 0$ (since $V_0 = 0$)

$\Rightarrow i_C(0^+) = -i_L(0^+) - i_R(0^+) = -(-12.25\,mA) = 12.25\,mA.$

Then,

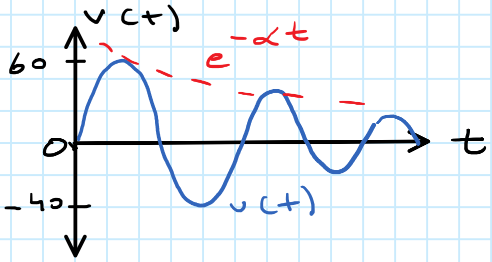
$$\frac{dv(0^+)}{dt} = \frac{1}{C} i_C(0^+) = \frac{(12.25)(10^{-3})}{(0.125)(10^{-6})} = 98000 \ \frac{V}{s}.$$

c-) To find B_1 and B_2:

$v(0^+) = V_0 = B_1 = 0.$, $\frac{dv(0^+)}{dt} = 98000 = -\alpha \cancel{B_1}^{\,0} + \omega_d B_2$

$\Rightarrow B_2 \cong 100.$

$\Rightarrow v(t) = 100\, e^{-200t} \sin(979.8t)$



— α: damping factor

$\alpha = \frac{1}{2RC}$

If R is ↑

α is small

It takes longer to damp.

— The oscillation frequency is ω_d.

$\omega_d = \sqrt{\omega_0^2 - \alpha^2}$, for a large R ↑, α is small ↓

$\Rightarrow \omega_d \approx \omega_0$ (resonant radian frequency.)

— Note that oscillators or sine wave generators are based on RLC circuit underdamped response.

$\omega_0 = \frac{1}{\sqrt{LC}}$ $\Rightarrow$ L, C determines the freq. of oscillation

Critically Damped Voltage Response:

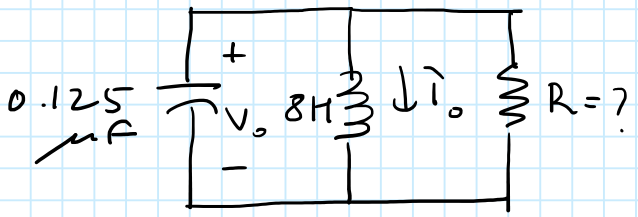
$S_1 = S_2 = -\alpha = \frac{-1}{2RC}$

The response is $v(t) = D_1 t e^{-\alpha t} + D_2 e^{-\alpha t}$

I.C's: $v(0^+) = D_2$, $\frac{dv(0^+)}{dt} = \frac{I_C(0^+)}{C} = D_1 - \alpha D_2$

Ex:

For the circuit given below, $V_0 = 0$, $I_0 = -12.25\,mA$



+

v

−

a-) Find the value for R such that the response is critically damped.

b-) Find v(t).

Ans:

a-) $\omega_0 = \dfrac{1}{\sqrt{LC}} = \sqrt{\dfrac{10^6}{8(0.125)}} = 10^3$ rad/s.

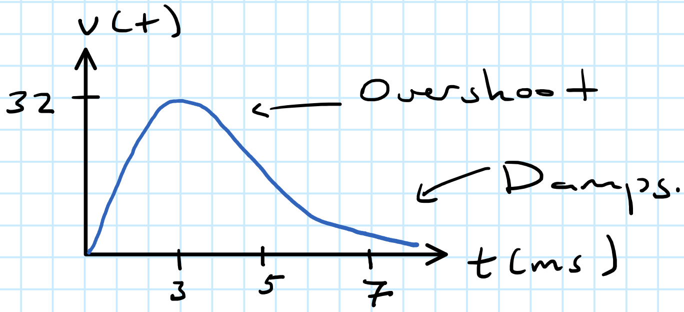
In critically damped response, $\alpha^2 = \omega_0^2$, $\alpha = \omega$

$\alpha = 10^3 = \dfrac{1}{2RC} \Rightarrow R = \dfrac{10^6}{(2000)(0.125)} = 4\,k\Omega.$

b-) $v(0^+) = 0 = D_2$

$\dfrac{dv(0^+)}{dt} = 98000\ \dfrac{V}{s} \Rightarrow D_1 = 98000\ \dfrac{V}{s}.$

$\Rightarrow v(t) = 98000\,t\,e^{-1000t}$ (V), $t \geq 0$



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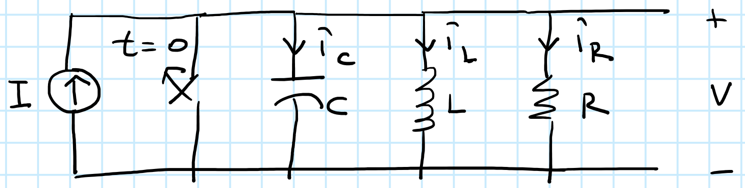
Step Response of Parallel RLC-Circuit: (Charging)

→ Consider the following circuit:



The governing equation for this circuit is:

$$\boxed{\frac{d^2 i_L}{dt^2} + \frac{1}{RC}\frac{di_L}{dt} + \frac{i_L}{LC} = \frac{I}{LC}}$$

- The solution can be expressed as i_L as a function of t.

Then, we have from $v = L\frac{di_L}{dt}$, differentiating both sides,

$$\frac{dv}{dt} = L\cdot\frac{d^2 i_L}{dt^2}$$

- Thus, the circuit equation is:

$$\frac{1}{L}\frac{dv}{dt} + \frac{1}{RC}\frac{v}{L} + \frac{1}{L^2C}\int_0^t v\,d\tau = \frac{I}{LC}$$

- Multiply both sides by LC,

$$\frac{1}{L}\int_0^t v\,d\tau + \frac{v}{R} + C\frac{dv}{dt} = I,$$

- Differentiate both sides w.r.t. t:

$$\frac{v}{L} + \frac{1}{R}\frac{dv}{dt} + C\frac{d^2v}{dt^2} = 0$$

or

$$\boxed{\frac{d^2v}{dt^2} + \frac{1}{RC}\frac{dv}{dt} + \frac{v}{LC} = 0}$$ (Circuit eqn. in terms of v)

→ The solution is:

$v = A_1 e^{s_1 t} + A_2 e^{s_2 t}$ (Overdamped)

or $v = B_1 e^{-\alpha t}\cos(\omega_d t) + B_2 e^{-\alpha t}\sin(\omega_d t)$ (Underdamped)

or $v = D_1 t e^{-\alpha t} + D_2 e^{-\alpha t}$, depending on the roots

s_1 and s_2 of the characteristic equation.

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→ The solution for i_L is:

$$i_L = I + A_1' e^{s_1 t} + A_2' e^{s_2 t} \quad \text{(Overdamped)}$$

or $$i_L = I + B_1' e^{-\alpha t} \cos(\omega_d t) + B_2' e^{-\alpha t} \sin(\omega_d t) \quad \text{(Underdamped)}$$

or $$i_L = I + D_1' t e^{-\alpha t} + D_2' e^{-\alpha t} \quad \text{(Critically damped)}$$

Ex:

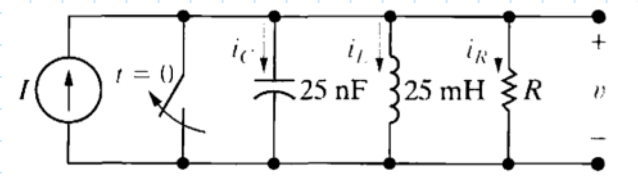
The initial energy stored in the circuit is zero. At t=0 a DC current source of 24 mA is applied to the circuit. The value of the resistor is 400Ω.

a-) What is the initial value of i_L ?

b-) " " " " " " $\frac{di_L}{dt}$?

c-) Find s_1, s_2.

d-) Find the expression for $i_L(t)$ for $t \geq 0$.



Ans:

a-) $i_L(0) = 0 \Rightarrow i_L(0^+) = 0$

b-) $v = L\frac{di_L}{dt} \Rightarrow \frac{di_L}{dt} = \frac{V}{L}$ and $v(0) = v(0^+) = 0$ from the cap. voltage.

$$\Rightarrow \frac{di_L(0^+)}{dt} = 0.$$

c-) $$\omega_0^2 = \frac{1}{LC} = \frac{10^{12}}{(25)(25)} = 16 \times 10^8$$

and

$$\alpha = \frac{1}{2RC} = \frac{10^9}{2(400)25} = 5 \times 10^4 \text{ rad/s.}$$

or $$\alpha^2 = 25 \times 10^8$$

Because, $\omega_0^2 < \alpha^2$, the roots of the characteristic equation are real and distinct. Thus,

$$s_1 = -5 \times 10^4 + 3 \times 10^4 = -20\,000 \text{ rad/s.}$$

$$s_2 = -5 \times 10^4 - 3 \times 10^4 = -80\,000 \text{ rad/s.}$$

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d-) The inductor current response is overdamped. Thus,

$$i_L = I + A_1' e^{s_1 t} + A_2' e^{s_2 t}$$

To find A_1' and A_2':

$$i_L(0) = I + A_1' + A_2' = 0$$

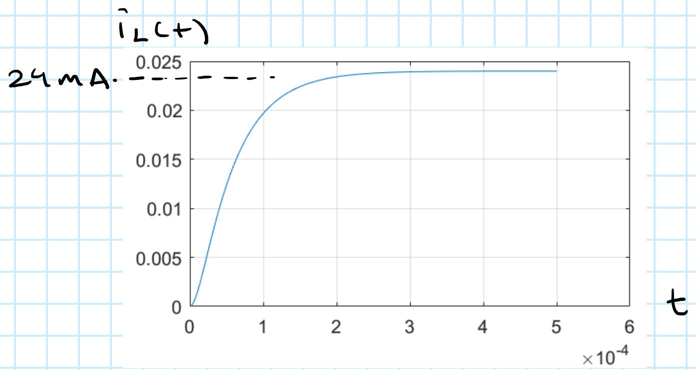
and

$$\frac{di_L(0)}{dt} = s_1 A_1' + s_2 A_2' = 0$$

Solving these equations yields $A_1' = -32\,mA$, $A_2' = 8\,mA$.

→Then, the solution for $i_L(t)$ is:

$$i_L(t) = (24 - 32e^{-20000t} + 8e^{-80000t})\,mA, \quad t \geq 0.$$



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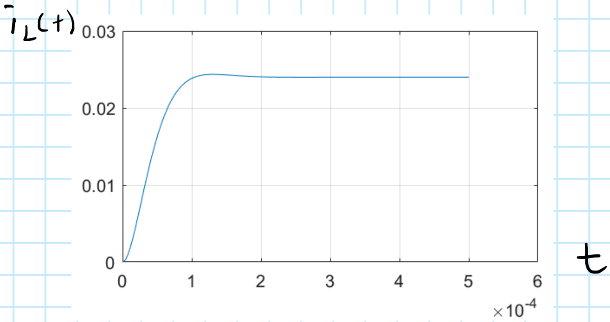
If R was 625Ω, then

$$i_L(t) = [24 - 24e^{-32000t}\cos(24000t) - 32e^{-32000t}\sin(24000t)]\,mA.$$



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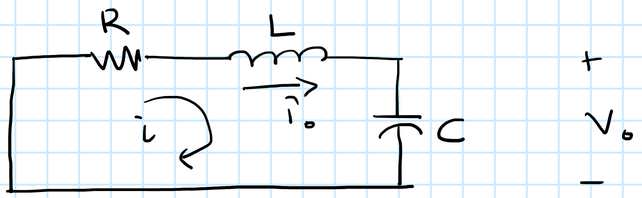
Natural and Step Response of Series RLC-Circuit:

Consider the circuit,



- The governing equation is : $Ri + L\frac{di}{dt} + \frac{1}{C}\int_0^t i\,d\tau + V_o = 0$
- Differentiate both sides :

$$R\frac{di}{dt} + L\frac{d^2 i}{dt^2} + \frac{i}{C} = 0$$

- Re-arranging,

$$\boxed{\frac{d^2 i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC} = 0}$$

→ The characteristic eqn is: $s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$

where $\boxed{s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2}}$ where $\boxed{\alpha = \frac{R}{2L}}$, $\boxed{\omega_o = \frac{1}{\sqrt{LC}}}$

and the solution of the natural response is :

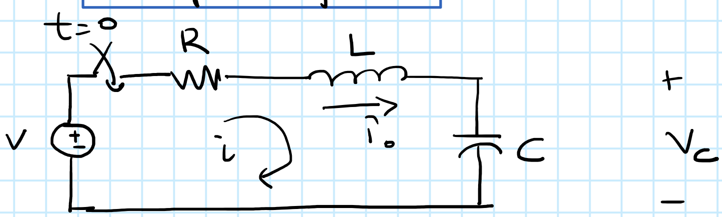
$i(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$, overdamped.

or $i(t) = B_1 e^{-\alpha t}\cos(\omega_d t) + B_2 e^{-\alpha t}\sin(\omega_d t)$, underdamped.

or $i(t) = D_1 t e^{-\alpha t} + D_2 e^{-\alpha t}$, critically damped responses.

The initial conditions can be obtained as before

To find the step response, we consider the following circuit:



- The governing eqn is : $\frac{d^2 V_c}{dt^2} + \frac{R}{L}\frac{dV_c}{dt} + \frac{V_c}{LC} = \frac{V}{LC}$

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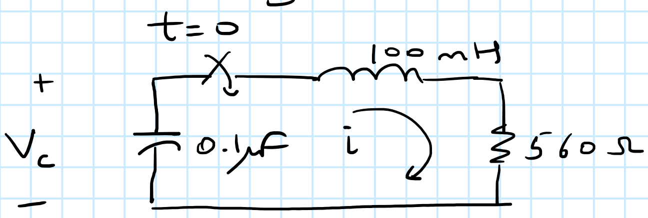
– The solution is:

$$\boxed{\begin{array}{ll} V_c = V_f + A_1' e^{s_1 t} + A_2' e^{s_2 t} & \text{(overdamped)} \\ \text{or} \quad V_c = V_f + B_1' e^{-\alpha t} \cos(\omega_d t) + B_2' e^{-\alpha t} \sin(\omega_d t) & \text{(under-damped)} \\ \text{or} \quad V_c = V_f + D_1' t e^{-\alpha t} + D_2' e^{-\alpha t} & \text{(critically damped.)} \end{array}}$$

Ex:

For the given circuit,



a-) Find $i(t)$, $t \geq 0$

b-) Find $V_c(t)$, $t \geq 0$.

$V_c(0) = 100\,V$.

Ans:

$$\omega_0^2 = \frac{1}{LC} = \frac{1}{(0.1)(0.1)\times 10^{-6}} = 10^8 \text{ rad/s.}$$

and

$$\alpha = \frac{R}{2L} = \frac{560}{2(0.1)} = 2800 \text{ rad/s.}$$

$\Rightarrow \quad \omega_0^2 > \alpha^2 \quad \Rightarrow$ underdamped!.

$\Rightarrow$ The solution is:

$$i(t) = B_1 e^{-\alpha t} \cos(\omega_d t) + B_2 e^{-\alpha t} \sin(\omega_d t)$$

where $\omega_d = \sqrt{\omega_0^2 - \alpha^2} = \left[10^8 - (2800)^2\right]^{\frac{1}{2}} = 9600$ rad/s.

and B_1 and B_2 can be obtained from:

$i(0) = 0 = B_1$.

For B_2, we need $\frac{di(0^+)}{dt}$, so $L\frac{di(0^+)}{dt} = V_0$

or $\frac{di(0^+)}{dt} = \frac{V_0}{L} = \frac{100}{0.1} = 1000 \frac{A}{s}$.

$$\frac{di(0)}{dt} = B_1(-\alpha) + B_2 \omega_d$$

$$\Rightarrow \frac{di(t)}{dt} = B_2(-\alpha)e^{-\alpha t}\sin(\omega_d t) + B_2 e^{-\alpha t} \omega_d \cos(\omega_d t)$$

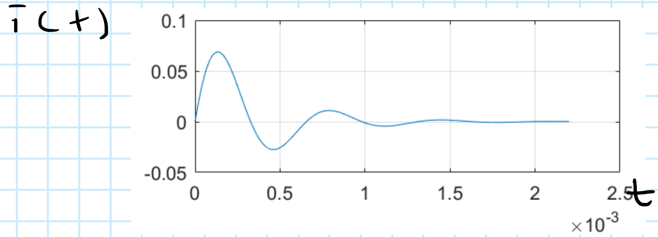
(with $B_1 = 0$)

Now,

$$\frac{di(0^+)}{dt} = 1000 = B_2 \omega_d \quad \Rightarrow B_2 = \frac{1000}{\omega_d} = \frac{1000}{9600} \approx 0.1042\,A$$

Thus,

$$i(t) = 0.1042 e^{-2800 t} \sin(9600 t), \quad t \geq 0.$$



(Underdamped)

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To find $V_c(t)$, we use the KVL in the circuit:

$$V_c = iR + L\frac{di}{dt} \quad \text{or} \quad V_c = -\frac{1}{C}\int_0^t i(\tau)\,d\tau + 100$$

C_1

Then,

$$V_c(t) = R\,i(t) + L\,\frac{di(t)}{dt}$$

$$= 560\left[0.1042e^{-2800t}\sin(5600t)\right]$$

$$+ 0.1\left[(0.1042)(-2800)e^{-2800}\sin(5600t) + (0.1042)e^{-2800}(5600)\cos(5600t)\right]$$

Re-arranging this expression gives us,

$$V_c(t) = \left[100\cos(5600t) + 25.17\sin(5600t)\right]e^{-2800t}, \quad t \geq 0.$$

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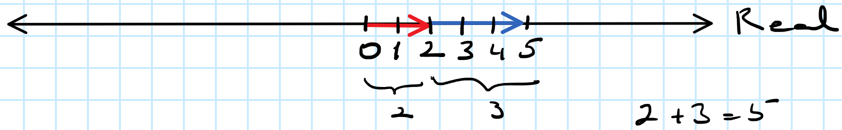
Complex Numbers:

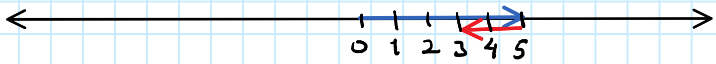
We show real numbers on the horizontal axis:

by addition, traslate on real axis.



$2+3=5$

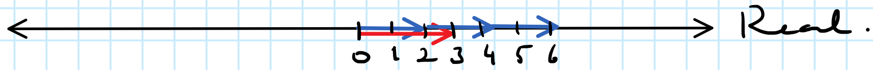
$\Rightarrow$ Substraction is also a traslation



$5-2=3$.

on the opposite direction.

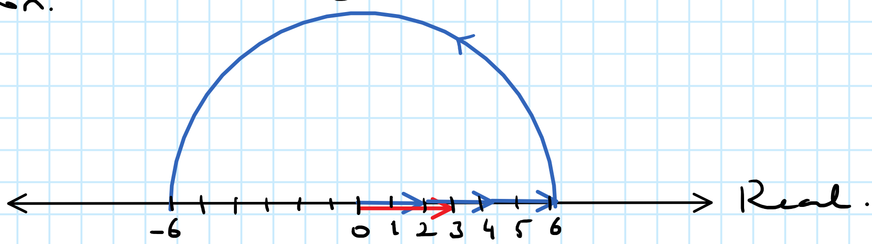
Multiplication: scaling the number.



2×3

↑ is scaled by 3.

Multiplication by a "−" number: yields scaling + rotation.



$2\times(-3)=-6$ (scaling + rotation)

Similarly, $(-6)\times(-1)$ means scaling + rotation.

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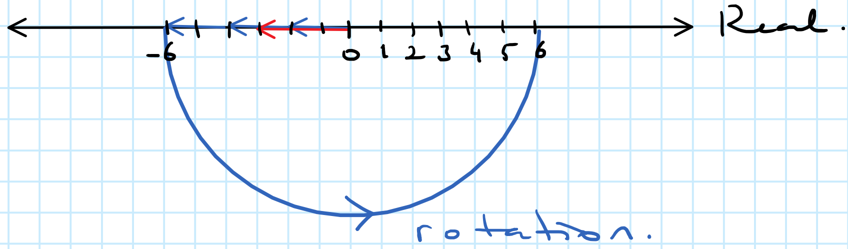
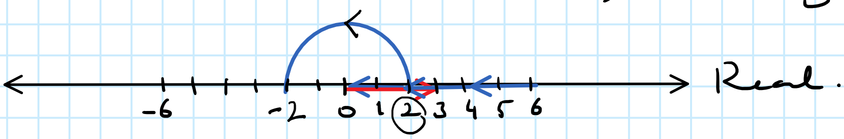
$(-6) \times (-1) = 6$



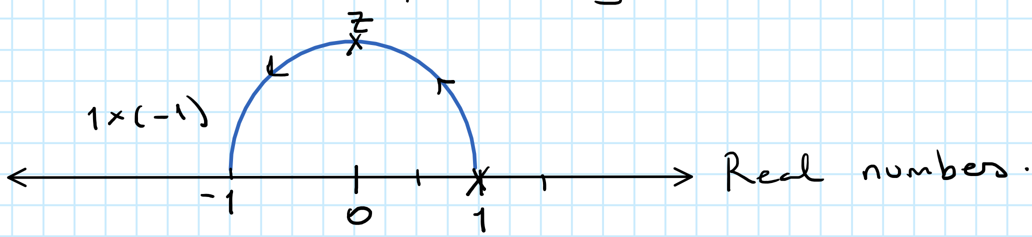
Division = Multip:

$6 \div (-3) = -2$

$\Rightarrow$ Scaling + rotation.



Let us consider the following case:



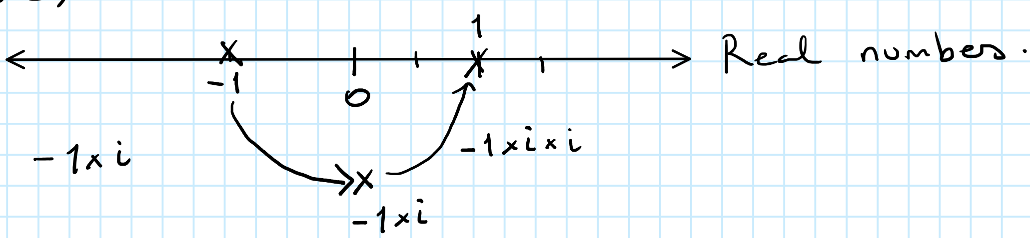
$1 \times z \times z = -1$, i = the number that makes a quarter turn.

$z^2 = -1$

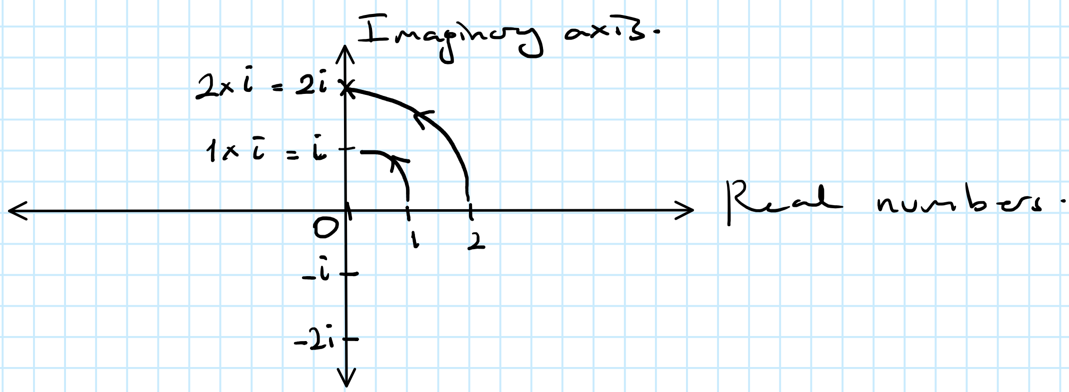
$z = \sqrt{-1} = i$ (Mathematicians)

$= j$ (Engineers)

For example,



Now,



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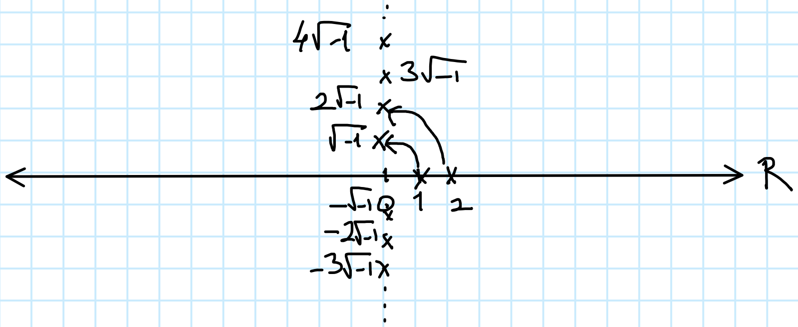
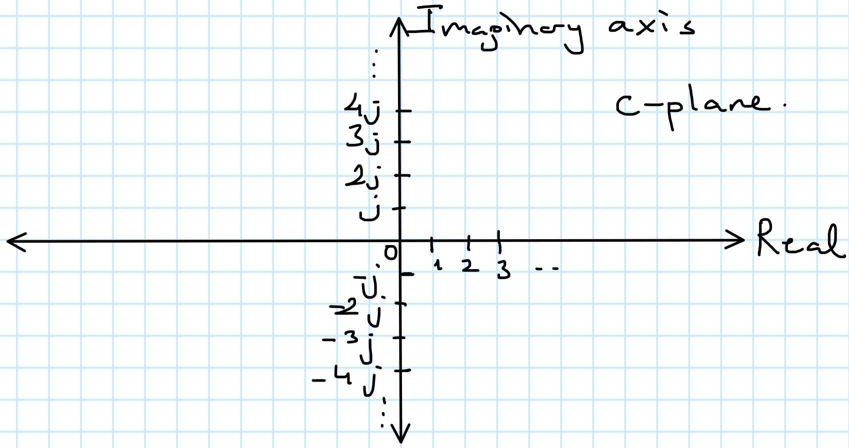
$\Rightarrow$ $z = j = \sqrt{-1}$, where is this number on the number line

$1 \times \sqrt{-1} = \sqrt{-1}$

$2 \times \sqrt{-1} = 2\sqrt{-1}$



$\Rightarrow$ We can draw a vertical line.

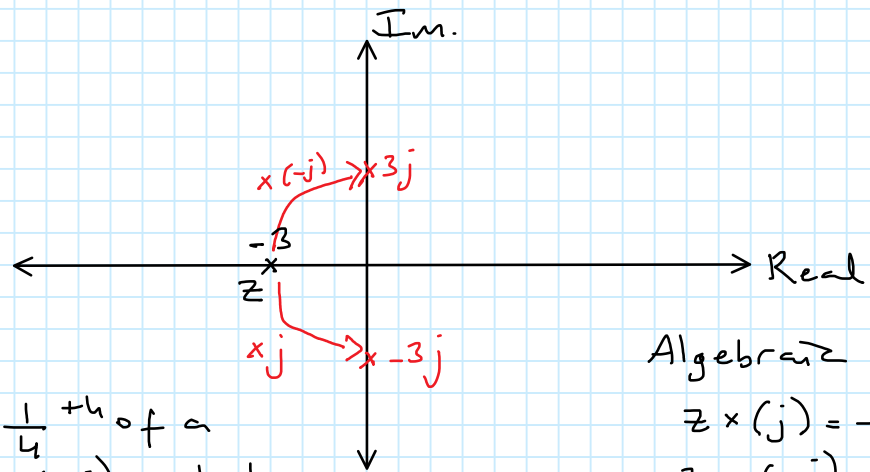


- This plane is called "complex plane" (c-plane) and any number on this plane is called a "complex number"

Ex

Suppose $z = -3$, what should be done to rotate this number $\frac{1}{4}$ of a turn cw and ccw?

Ans



multiply

$\Rightarrow$ $\times (j)$ rotates $\frac{1}{4}$th of a turn ccw. $\times(-j)$ rotates $\frac{1}{4}$th of a turn cw.

Algebraic calculations:

$z \times (j) = -3j$

$z \times (-j) = 3j$

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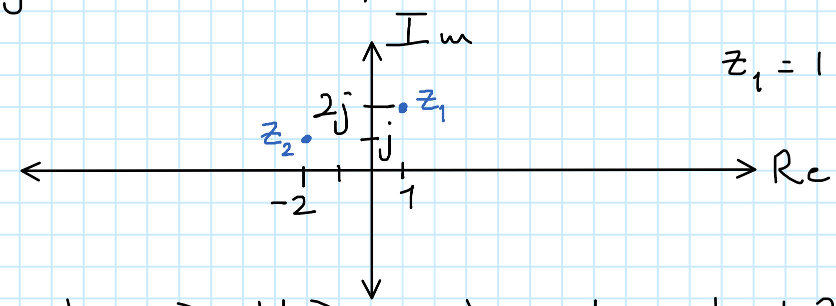
Writing a Complex Number in terms of its Real and imaginary parts (components):



$z_1 = 1 + 2j$

$z_2 = z_1 \times j = ?$ where is this number located?

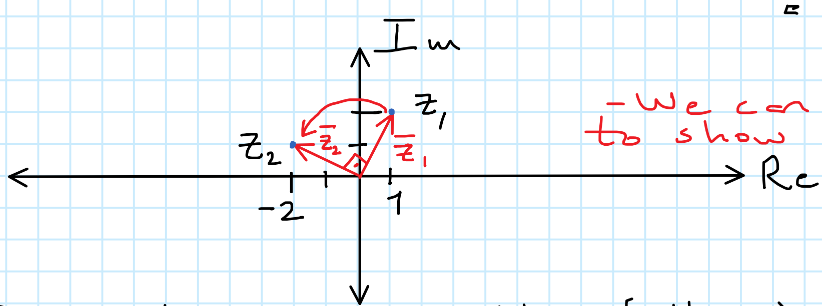
$j = \sqrt{-1}$

$j^2 = -1$

Ans

Algebraic solution: $z_1 \times j = (1 + 2j) \times (j) = j + 2j^2$

$= -2 + j$



- We can use a vector to show complex numbers

⇒ Up to this point, we have the following notations:

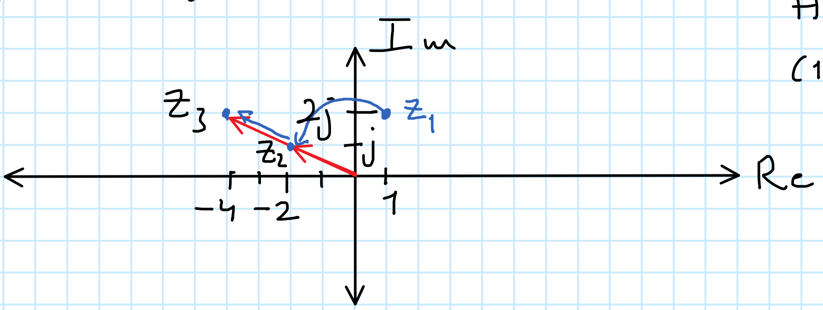
$z = \underbrace{a}_{\text{Real}} + \underbrace{jb}_{\text{Imaginary}}$ (component notation)

$\bar{z} = \bar{a} + j\bar{b}$ (vector notation, used in geometric analysis)

—o—

What if we multiply z_1 by $2j$.

$z_3 = z_1 \times (2j) = ?$



Algebraic Sol.:

$(1 + 2j) \cdot (2j)$

$= 2j + 4j^2$

$z_3 = -4 + 2j$

⇒ $z_1 \times (2j) = $ (rotation + scaling by 2.)

—o—

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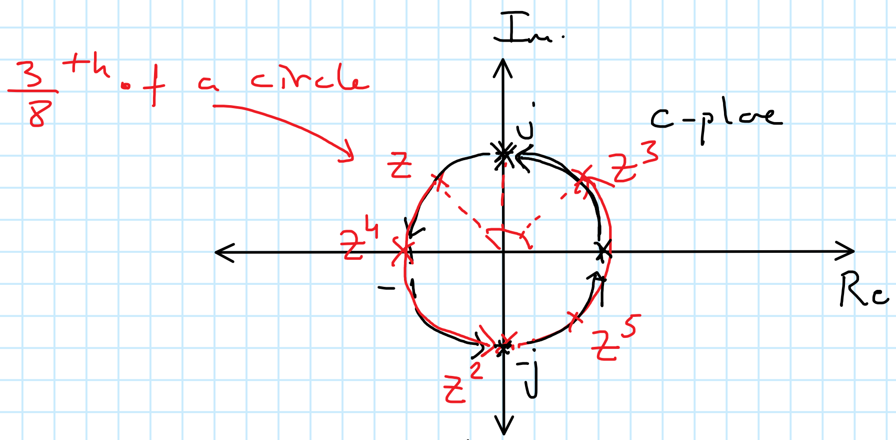
Exponentiation with Complex Numbers (Power of a Complex Number):

$j^n = ?$



$1 \times j = j \quad (j^1)$

$j \times j = -1 \quad (j^2)$

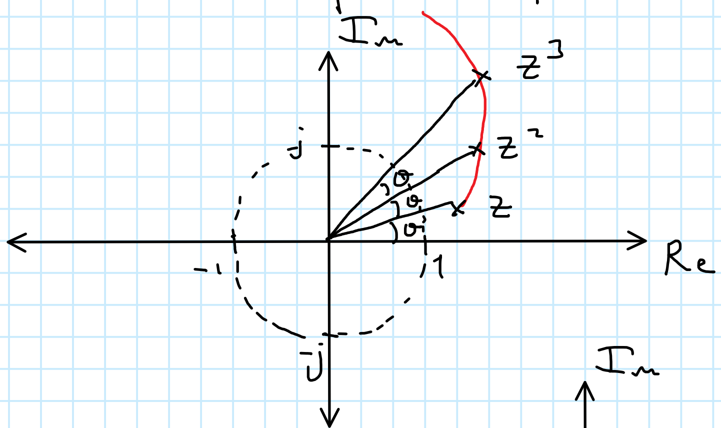
$-1 \times j = -j \quad (j^3)$

$-j \times j = 1 \quad (j^4)$

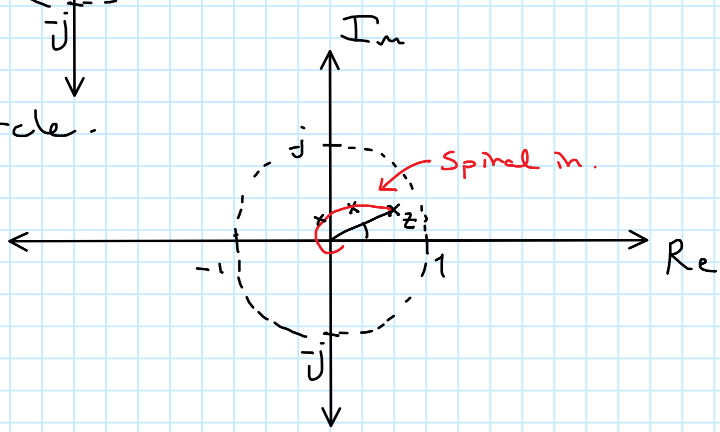
$1 \times j = j \quad (j^5)$

$\Rightarrow$ Taking the power of (j) rotates ccw on the unit circle

- Taking the power of any complex number z which is on the unit circle rotates ccw by the same angle that z makes with the real axis
- What if z is outside the unit circle? Then, what happens when we take the power of z



- If z is inside the unit circle.



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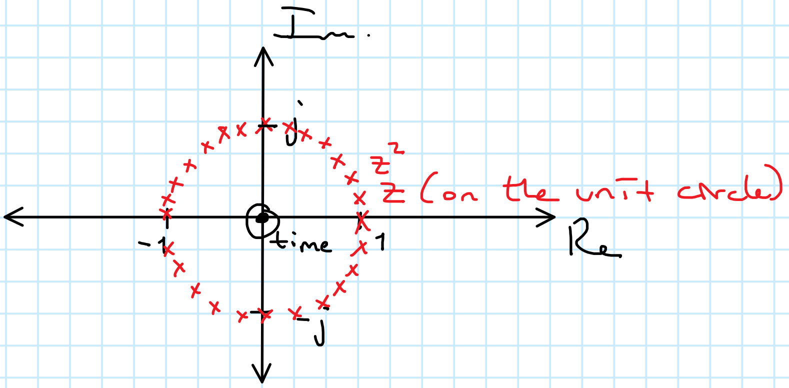
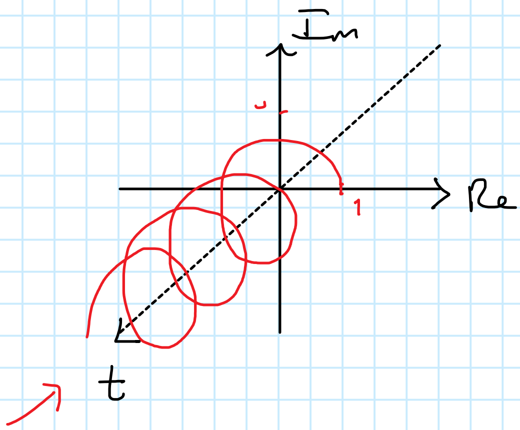
Time Variable:

Let us introduce time variable in complex domain.
Let us assume that we increase the exponent over time.



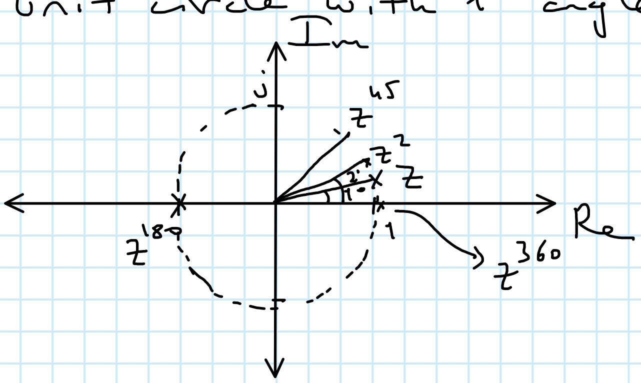
In 3D view:



- In 3D, with the time axis, we get a helix shape.

—o—

If z is on the unit circle with 1° angle.



z with 1° is not a base or a standart

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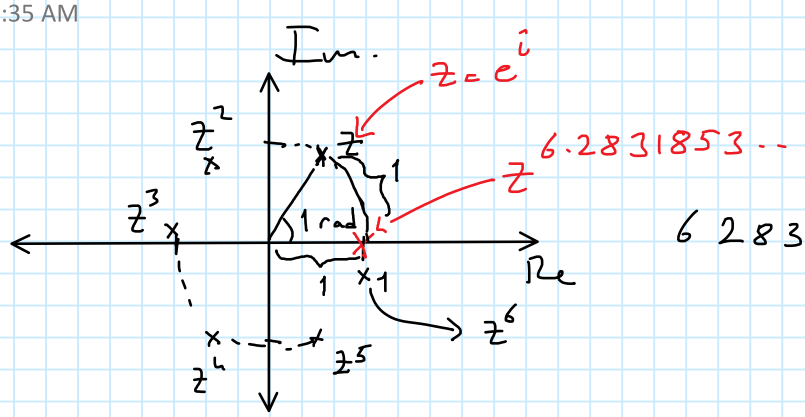
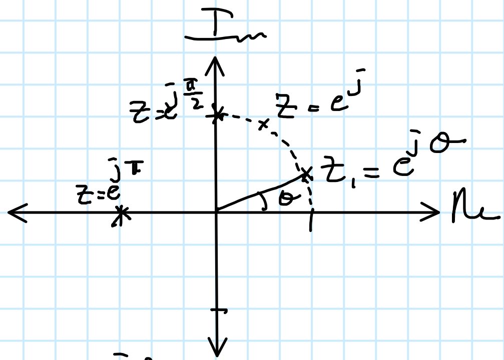
The base is



6 2 8 3 1 $= 2\pi$

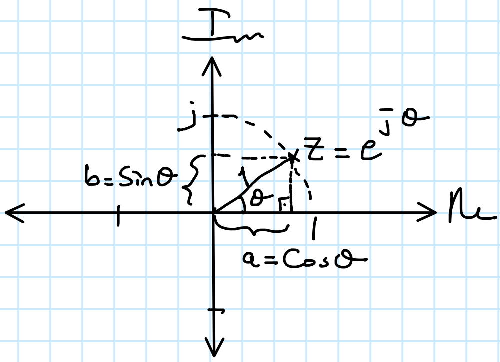
- The z is also equal to $\boxed{z = e^{j}}$, $e = 2.71828\ldots$ (Euler's number)

- Also, $z_1 = e^{j\theta}$ is on the unit circle with angle θ in radians



$z = e^{j3.14} \approx \pi$

Let us observe $z = e^{j\theta}$ closely: (complex exponential)



$$\Rightarrow \quad \underbrace{z = e^{j\theta}}_{\text{Exponential notation}} = \underbrace{a + jb}_{\text{Rectangular notation}} = \cos\theta + j\sin\theta \quad \text{(Euler's formula)}$$

$$z = re^{j\theta} = r\cos\theta + jr\sin\theta$$

Ex:

Find a $\frac{1}{4}$ th turn of a complex number ($z = 1$) on the unit circle in terms of exponential notation.

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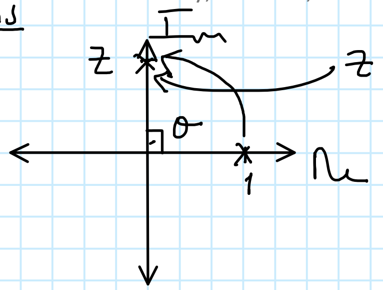
Ans



$z = e^{j\theta}$ $(\theta = \frac{\pi}{2}) \Rightarrow z = e^{j\frac{\pi}{2}} = j.$

Proof.

$$e^{j\frac{\pi}{2}} = \underbrace{\cos\frac{\pi}{2}}_{0} + j\underbrace{\sin\frac{\pi}{2}}_{1} = j$$

—o—

Ex:

Draw $z = e^{j2\pi f t}$ (Exponential notation)

where f = frequency = constant , t = time = variable.

Ans

Analysis

e = constant = 2,7 .

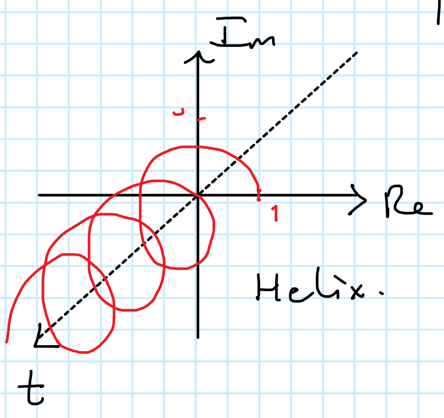
j = constant

2π = constant

f = constant

$\Rightarrow z = e^{j\theta}$

$\theta = 2\pi f t$

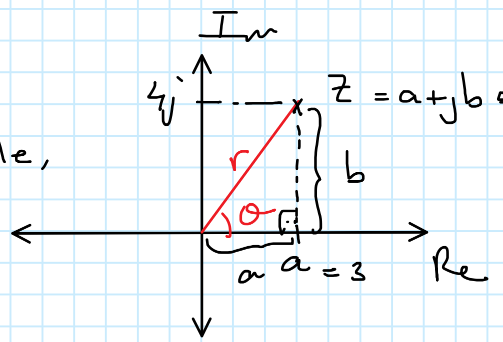


—o—

- Until now, The formats for complex numbers.

$z = a + jb$ = Rectangular , $\vec{z}$ = vectorial,

3rd format. Polar form (exponential form)



$z = a + jb = 3 + 4j = r e^{j\theta} = r\angle\theta$

where

r = Magnitude, amplitude, radius, modulus.

r = distance from origin

θ = Argument, Angle, Phase

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$r = |z|$ (r is found by taking the absolute vale of z)

$\|z\| = |z| =$ Magnitude $= r$

Transformations

A complex number given in rectangular form

$z = a + jb \quad , \quad z = re^{j\theta}$

Rect. to polar $\Rightarrow$ $r^2 = a^2 + b^2 \quad , \quad \theta = \tan^{-1}\left(\frac{b}{a}\right)$

Polar to Rect $\Rightarrow$ $a = r\cos\theta, \quad b = r\sin\theta.$

—o—

Addition & Subtraction of Complex Numbers:

– Using rectangular notation is easier

Ex:

$z_1 = 3 + 4i$

$z_2 = 2 - 2i$

a) $z_1 + z_2 = ?$

b-) $z_1 - z_2 = ?$

Ans

$z_1 + z_2 = (3 + 4i) + (2 - 2i)$

$= (3 + 2) + (4i - 2i)$

$= 5 + (4 - 2)i = 5 + 2i$

$z_1 - z_2 = (3 + 4i) - (2 - 2i)$

$= (3 + 4i) - 2 + 2i$

$= (3 - 2) + (4i + 2i)$

$= 1 + 6i$

Multiplication & Division

Polar form is easier.

Ex

$z_1 = 3 + 4j, \quad z_2 = 8 - 6j$

a-) $z_1 \cdot z_2 = ?$

b-) $\frac{z_2}{z_1} = ?$

Ans

$z_1 \cdot z_2 = (3 + 4j) \cdot (8 - 6j)$

$= 24 - 18j + 32j - 24j^2$

$= 24 - 18j + 32j + 24$

$= 48 + 14j$

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$z_1 = 3 + 4j, \quad z_2 = 8 - 6j$

$z_1 = 5e^{j\tan^{-1}\frac{4}{3}}, \quad z_2 = 10e^{j\tan^{-1}\left(\frac{-6}{8}\right)}$

$z_1 \cdot z_2 = 5e^{j0.93} \cdot 10e^{-j0.64} = 50e^{j(0.93-0.64)} = 50e^{j0.29}$

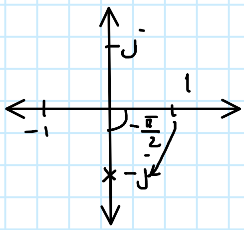
$= 50\cos(0.29) + j50\sin(0.29) = 48 + j14$ ✓

b-) $\dfrac{z_2}{z_1} = \dfrac{8-6j}{3+4j} \cdot \dfrac{(3-4j)}{(3-4j)} = \dfrac{\cancel{24} - 32j - 18j - \cancel{24}}{9+16} = \dfrac{-50j}{25} = -2j$

Conjugate (reverse the sign of the imaginary part)

$\dfrac{z_2}{z_1} = \dfrac{10e^{j(-0.64)}}{5e^{j0.93}} = \dfrac{10}{5} \cdot e^{j(-0.64-0.93)}$

$= 2e^{j(-1.57)} = 2\underbrace{e^{j\left(-\frac{\pi}{2}\right)}}_{-j} = -2j.$ ✓



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Friday, December 17, 2021 9:33 AM

- Sinusoidal Steady State Analysis - (Cp.5)

- Sinusoidal Source:

Consider the following voltage source:

$$v = V_m \cos(\omega t + \phi) + V_{off}$$

where

t = time variable (s)

v = Voltage (V)

ω = Radian frequency (rad/s)

V_m = Maximum amplitude (V)

ϕ = Phase angle (rad)

and V_{off} = DC voltage added to v. (offset)

$\omega = 2\pi f$, f = frequency. (Hz)

and $T = \frac{1}{f}$ = Period (s)

- The phasor is a complex number that carries the amplitude and phase information of a sinusoidal function.
- It is based on the Euler's Identity

$$e^{\mp j\theta} = \cos\theta \mp j \sin\theta .$$

- Before moving further, let us study complex numbers!

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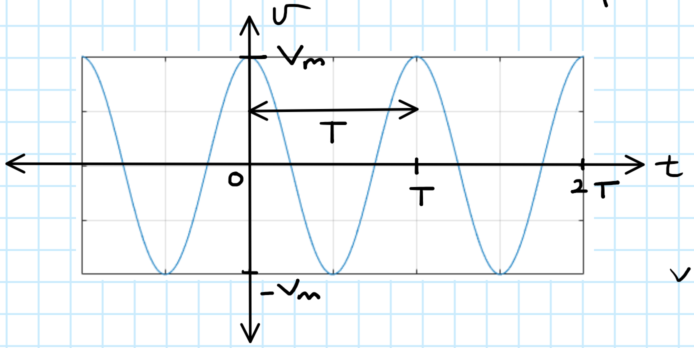
Friday, December 4, 2020 11:51 AM

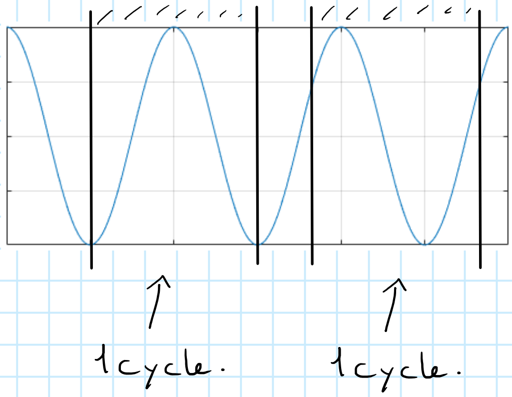
If we plot a sinusoidal source waveform w.r.t. time:



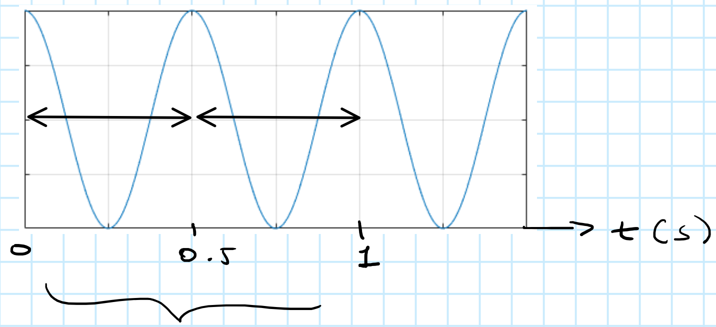
$v(t) = v(t+T)$
(Periodic)

Thus,

Cycle: The smallest non-repeatable portion of the periodic waveform.

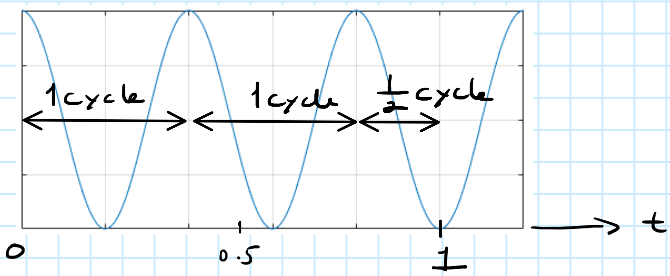


Frequency: # of cyles in 1 sec.



There are 2 cycles ⇒ f = 2 Hz.

or



$$f = 1 + 1 + \frac{1}{2} = 2 + \frac{1}{2} = \frac{5}{2} \text{ Hz} = 2.5 \text{ Hz}.$$

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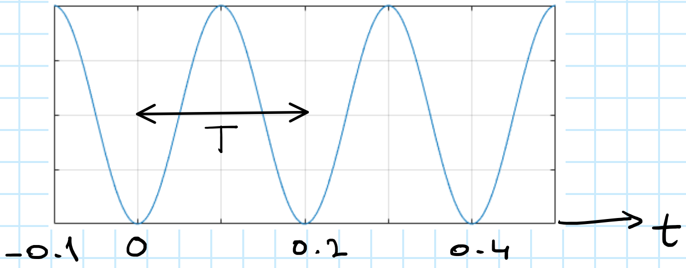
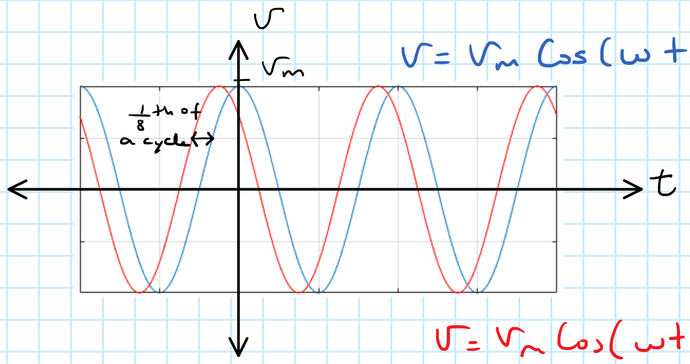
Period = T = Time for 1 cycle.



$\Rightarrow T = 0.2$ sec. $\Rightarrow \boxed{f = \frac{1}{T}} = \frac{1}{0.2} = 5$ Hz.

Phase Angle = ϕ

It shifts the graph left or right for + or − ϕ respectively.



$v = V_m \cos(\omega t)$

$v = V_m \cos(\omega t + \frac{\pi}{4})$

$\phi = \frac{\pi}{4}$

− 1 cycle is from 0 to 2π.

$\Rightarrow \frac{\pi}{4}$ is $\frac{1}{8}$th of a cycle.

That's why we shift the graph of $v = V_m \cos(\omega t)$ by a $\frac{1}{8}$th of a cycle to the left.

The Radian Frequency, ω :

Consider

$$v(t) = V_m \cos(\omega t + \phi) \quad V.$$

$\uparrow ?$

For $\phi = 0$ for simplicity

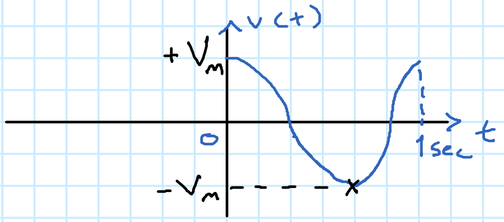
$$v(t) = V_m \cos(\underbrace{\omega t}_{\text{Angle (radian)}})$$

Put $\omega = 0$:

$$v(t) = V_m. \quad \text{(DC signal)}$$

for $\omega = 2\pi$.

$$v(t) = V_m \cos(2\pi t)$$



$t = 0 \rightarrow v(0) = V_m$

$t = 0.5 \rightarrow v(0.5) = -V_m$

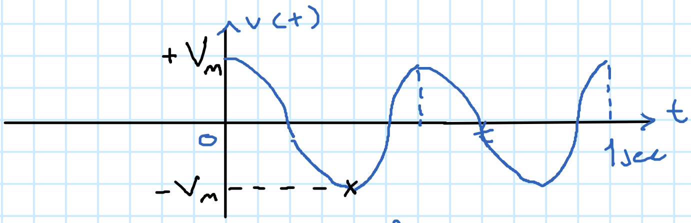
$t = 1 \text{ sec} \rightarrow v(1) = V_m$

$\Rightarrow f = 1 \text{ Hz}.$

For $t = 0 \rightarrow 1$ Angle $= \omega t \rightarrow 0 - 2\pi$.

For $\omega = 4\pi$.

$t = 0 \rightarrow 1$ Angle $= \omega t \rightarrow 0 - 4\pi$



$f = 2 \text{ Hz}$

- Therefore, ω = radian freq. gives the number of cycles in 1 sec in terms of radians (# of 2π's).

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Monday, November 29, 2021 1:55 PM

Another important property of a sinusoidal voltage (or current) is the "rms value".

rms : Root Mean Square.

Thus, if $v(t) = V_m \cos(\omega t + \phi)$

$$\boxed{V_{rms} = \left[\frac{1}{T}\int_{t_0}^{t_0+T} v^2 dt\right]^{\frac{1}{2}}}$$

= Root of the mean value (average) of the squared voltage.

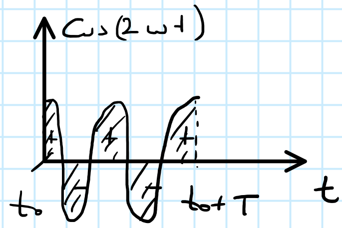
$$\Rightarrow V_{rms} = \left[\frac{1}{T}\int_{t_0}^{t_0+T} V_m^2 \cos^2(\omega t + \phi)\, dt\right]^{\frac{1}{2}} =$$

Using the identity $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$

$$V_{rms} = \left[\frac{1}{T} V_m^2 \underbrace{\int_{t_0}^{t_0+T} \frac{1}{2}\left[1 + \cos(2\omega t + 2\phi)\right] dt}\right]^{\frac{1}{2}}$$

$$\underbrace{\int_{t_0}^{t_0+T} \frac{1}{2} dt} + \underbrace{\frac{1}{2}\int_{t_0}^{t_0+T} \cos(2\omega t + 2\phi)\, dt}_{=0}$$

$$\frac{1}{2}\int_{t_0}^{t_0+T} dt = \frac{T}{2}.$$



$$V_{rms} = \left[\frac{1}{\cancel{T}} V_m^2 \frac{\cancel{T}}{2}\right]^{\frac{1}{2}} = \left(\frac{V_m^2}{2}\right)^{\frac{1}{2}} = \frac{V_m}{\sqrt{2}} \quad (V)$$

or

$$\boxed{V_{rms} = \frac{V_m}{\sqrt{2}} \quad (V)}$$

(rms value of a sinusoidal voltage signal $v(t)$)

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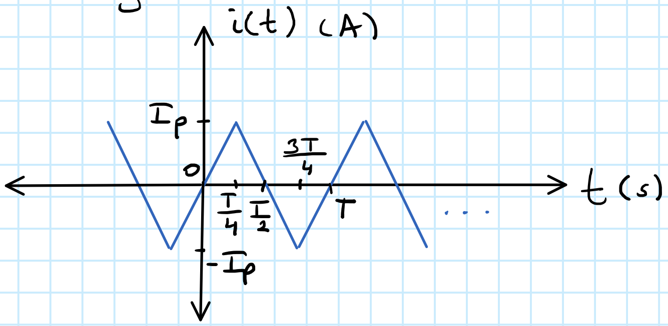
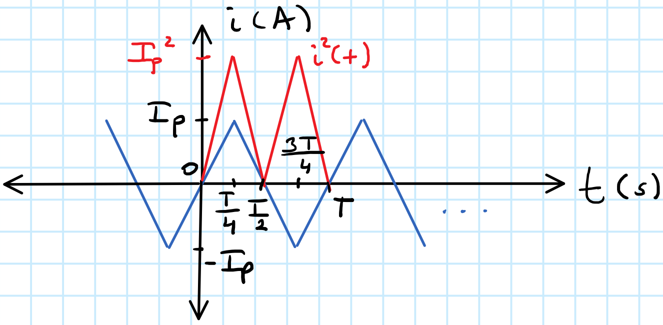
Ex:

Find the rms value of the periodic triangular function given below.



Ans:

$$i_{rms} = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} i^2\,dt}$$



Then,

$$\int_{t_0}^{t_0+T} i^2\,dt = 4\int_0^{T/4} i^2\,dt$$

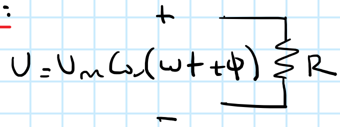
$i^2 \to i^2(t)$

From the graph:

$$i(t) = \underbrace{\frac{4I_p}{T}}_{\text{Slope}}\, t, \quad 0 < t < T/4$$

$$\Rightarrow 4\int_0^{T/4} i^2(t)\,dt = 4\int_0^{T/4} \underbrace{\frac{16\,I_p^2}{T^2}\,t^2}_{i^2(t)}\,dt = \frac{I_p^2 T}{3}, \Rightarrow i_{rms} = \sqrt{\frac{1}{\cancel{T}}\cdot\frac{I_p^2\cancel{T}}{3}} = \frac{I_p}{\sqrt{3}}.$$

– The use of rms value for sinusoidal voltage and/or current :



$P_R = ?$ $\quad P_R = U \cdot i = \frac{U^2}{R} = i^2 \cdot R \ (W)$

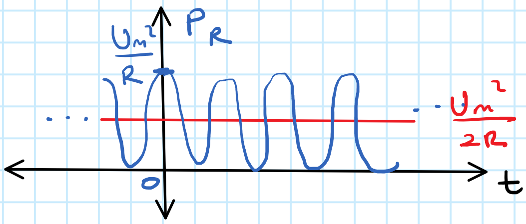
$$\Rightarrow P_R = \frac{U_m^2 \cos^2(\omega t + \phi)}{R} \ (W)$$

Using the identity $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$

$$P_R = \frac{U_m^2}{R}\frac{1}{2}\left[1 + \cos(2\omega t + 2\phi)\right] = \frac{U_m^2}{2R} + \frac{U_m^2}{2R}\cos(2\omega t + 2\phi)$$

Let's say $\phi = 0$ for simplicity.

$$P_R = \frac{U_m^2}{2R} + \frac{U_m^2}{2R}\cos(2\omega t)$$



– This power keeps changing. Thus, it is not useful.

– Thus, we use the average of this power.

$$P_{avg} = \frac{1}{T}\int_0^T P_R\,dt = \frac{1}{T}\left[\int_0^T \frac{U_m^2}{2R}\,dt + \underbrace{\int_0^T \frac{U_m^2}{2R}\cos(2\omega t + 2\phi)\,dt}_{=0}\right]$$

$$\Rightarrow \boxed{P_{avg} = \frac{1}{\cancel{T}}\frac{U_m^2}{2R}\cancel{T} = \frac{U_m^2}{2R} \ (W)}$$

$$\frac{V_{rms}^2 \cancel{2}}{\cancel{2}R} = \frac{V_{rms}^2}{R}$$

Ex :

A light bulb is connected to $V = 311\cos(\omega t)$ (V) where $\omega = 2\pi f = 2\pi(50) = 100\pi$. Find the average power consumed by $R = 100\,\Omega$ when connected to this voltage source.

Ans:

The power consumed means the average power P_{avg}.

$$\Rightarrow P_{avg} = \frac{V_m^2}{2R} = \frac{(311)^2}{2(100)} = 484 \ W.$$

—o—

– We could also use the relation $V_{rms} = \frac{V_m}{\sqrt{2}}$. Then,

$$P_{avg} = \frac{(\sqrt{2}V_{rms})^2}{2R} = \frac{\cancel{2}V_{rms}^2}{\cancel{2}R} = \frac{V_{rms}^2}{R} \ (W)$$

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Friday, December 11, 2020 10:34 AM

Thus, the same problem could be solved by finding the rms voltage first,

$$V_{rms} = \frac{V_m}{\sqrt{2}} = \frac{311}{\sqrt{2}} = 220\,V.$$

Then, for finding the average power, use

$$P_{avg} = \frac{V_{rms}^2}{R} = \frac{220^2}{100} = 484\,W.$$

Ex:

Find the resistance of a 60 W light bulb?

Ans:

$P = 60W = P_{avg}$.

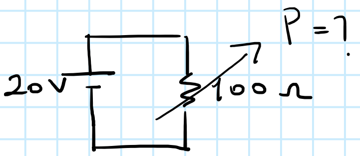
$$P_{avg} = \frac{V_m^2}{2R}$$

or

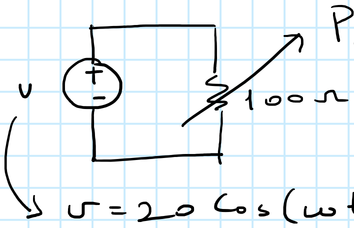
$$P_{avg} = \frac{V_{rms}^2}{R} \checkmark$$

$$\Rightarrow 60 = \frac{(220)^2}{R} \Rightarrow R = 806\,\Omega.$$

Ex:



$$P = \frac{20^2}{100} = \frac{2\cancel{0}\cdot 2\cancel{0}}{1\cancel{0}\cancel{0}} = 4\,W.$$

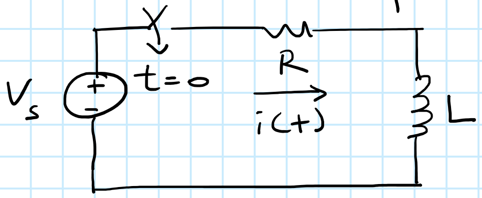


$$P_{avg} = ? = \frac{V_{rms}^2}{R} = \frac{(14.14)^2}{100} = \underline{\underline{2\,W}}$$

$v = \underbrace{20}_{V_m} \cos(\omega t)$, $V_{rms} = \frac{20}{\sqrt{2}} = 14.14V.$

Sinusoidal Response:

Consider the following circuit:



where $V_s = V_m \cos(\omega t + \phi)$

$i(0) = 0$

$i(t) = ?$, $t \geq 0$.

→ This is the step response of an RL-circuit as we've known before. The difference is that we have a sinusoidal source, V_s.

P81

Friday, December 11, 2020 10:51 AM

Writing the mesh equation (KVL):

$$L\frac{di}{dt} + Ri = V_m \cos(\omega t + \phi)$$

The solution is:

$$i = \frac{-V_m}{\sqrt{R^2+\omega^2L^2}} \cos(\phi+\theta)\, e^{-(\frac{R}{L}t)} + \frac{V_m}{\sqrt{R^2+\omega^2L^2}} \cos(\omega t+\phi-\theta)$$

where $\theta = \tan^{-1}\left(\frac{\omega L}{R}\right)$.

- The first term of this solution is called the "transient response".
- The 2^{nd} term of this solution is called the "Steady State Response".

- Because for $t \geq 0$, the 1^{st} term decreases rapidly, and the 2^{nd} term starts to dominate.
- So, the 1^{st} term disappears after a while, and the 2^{nd} term remains.
- We can make the following conclusions for the steady state response:

1-) It is also sinusoidal.

2-) It has the same frequency as the source.

3-) Amplitude and phase change.

4-) The solution of the differential equation is rather long and complicated.

→ The remedy for conclusion 4 is to use "Phasors".

Phasor = Complex number representation of the sinusoidal expressions to make mathematical computations easier.

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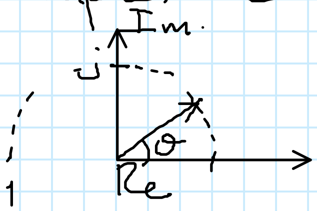
Monday, March 8, 2021 11:08 AM

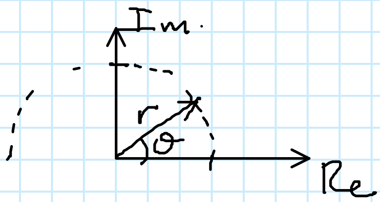
Let's start to use complex numbers:

Consider a complex exponential:

$$\bar{z} = e^{j\theta}$$



$$\bar{z} = r e^{j\theta}$$



- - - - -

$$\underbrace{z = re^{j\theta}}_{\text{Polar form}} = \underbrace{r\cos\theta + j r\sin\theta}_{\text{Rectangular}}$$

$$\Rightarrow \quad r\cos\theta = \mathrm{Re}[re^{j\theta}] \;, \quad r\sin\theta = \mathrm{Im}[re^{j\theta}]$$

Then, we can write the sinusoidal source

$V = V_m \cos(\omega t + \phi)$ as $V = \mathrm{Re}[V_m e^{j(\omega t + \phi)}]$

or in the sine case:

$$V = \mathrm{Im}[V_m e^{j(\omega t + \phi)}]$$

Considering the cosine notation

Phasor

$$\boxed{V = \mathrm{Re}[V_m e^{j(\omega t + \phi)}] = \mathrm{Re}[V_m e^{j\omega t} \cdot e^{j\phi}]} \quad \text{(Inverse formula)}$$

Since the frequency (ω) does not change in the solution for voltages and/or currents, we don't need "ωt" term in the computations, it can be neglected. Then, we will only use

$$V = \mathrm{Re}[V_m e^{j\phi} \, e^{j\omega t}]$$ where $\boxed{z = V_m e^{j\phi}}$ is called the "phasor" of V. or $\boxed{z = V_m \angle\phi}$

The phasor is a complex exponential having the amplitude and phase information of the source.

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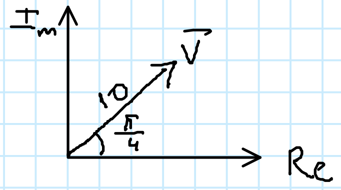
Ex: For $V = 10\cos(\underbrace{2000\pi t}_{X} + \frac{\pi}{4})$, write its phasor?

Ans:

$$\vec{V} = 10 e^{j\frac{\pi}{4}}$$

or $\vec{V} = 10 \angle \frac{\pi}{4}$



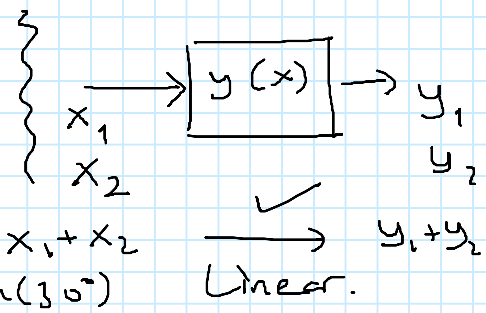
In calculations, using phasors instead of sinusoidals makes the solution easier.

Ex.

If $y_1 = 20\cos(\omega t - 30^\circ)$ and $y_2 = 40\cos(\omega t + 60^\circ)$

a-) $y_1 + y_2 = ?$ using sinusoidals

b-) Find $y_1 + y_2$ using phasors.



Ans:

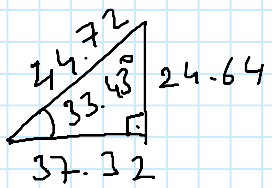
a-) $y_1 = 20\cos(\omega t)\cos(30^\circ) + 20\sin(\omega t)\sin(30^\circ)$

$y_2 = 40\cos(\omega t)\cos(60^\circ) - 40\sin(\omega t)\sin(60^\circ)$

$$y_1 + y_2 = \cos(\omega t)(20\cos 30^\circ + 40\cos 60^\circ) + \sin(\omega t)(20\sin 30^\circ - 40\sin 60^\circ)$$

$$= 37.32\cos(\omega t) - 24.64\sin(\omega t)$$

$$\Rightarrow y_1 + y_2 = 44.72\left[\underbrace{\frac{37.32}{44.72}}_{\cos(33.43^\circ)}\cos(\omega t) - \underbrace{\frac{24.64}{44.72}}_{\sin(33.43^\circ)}\sin(\omega t)\right]$$



From $\cos A\cos B - \sin A\sin B = \cos(A+B)$

$$\Rightarrow y_1 + y_2 = 44.72\cos(\omega t + 33.43^\circ)$$

b-) $\bar{y}_1 = 20 e^{-j\frac{\pi}{6}}$, $\bar{y}_2 = 40 e^{j\frac{\pi}{3}}$

$$\bar{y}_1 + \bar{y}_2 = \left[20\cos\left(-\frac{\pi}{6}\right) + j20\sin\left(-\frac{\pi}{6}\right)\right] + \left[40\cos\left(\frac{\pi}{3}\right) + j40\sin\left(\frac{\pi}{3}\right)\right]$$

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$$= (17.32 - j10) + (20 + j34.64) = 37.32 + j24.64 = 44.72\, e^{j0.5835}$$

$$\bar{y}_1 + \bar{y}_2 = 44.72\, e^{j33.43^\circ} \text{ (Phasor)}$$

Time expression of $\bar{y}_1 + \bar{y}_2$ is

$$y_1 + y_2 = Re\left[(\bar{y}_1 + \bar{y}_2) \cdot e^{j\omega t}\right] = Re\left[44.72\, e^{j33.43^\circ} \cdot e^{j\omega t}\right]$$

$$= Re\left[44.72\, e^{j\underbrace{(\omega t + 33.43^\circ)}_{\theta}}\right]$$

$$y_1 + y_2 = 44.72 \cos(\omega t + 33.43^\circ)$$ ✓

- Passive Circuit Elements (R, L, C) in $\underbrace{\text{Frequency Domain}}_{\text{Phasor domain}}$ -

1-) V-I Relation for a Resistor:

$$V = R\left[I_m \cos(\omega t + \phi)\right]$$
$$= R I_m \cos(\omega t + \phi) \text{ (V)}.$$

$$\Rightarrow \bar{V} = R I_m \cdot e^{j\phi} = R \underbrace{I_m \angle \phi}_{\bar{I}}$$

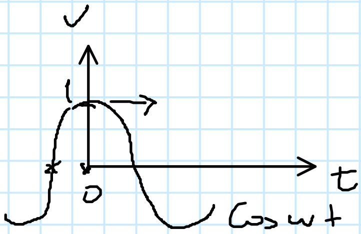
or $\boxed{\bar{V} = R\bar{I}}$

2-) V-I Relation for an Inductor.

$V = L \dfrac{di}{dt}$ assuming that $i = i(t) = I_m \cos(\omega t + \phi)$

or

$$V = L(-\omega) I_m \sin(\omega t + \phi)$$
$$= -\omega L I_m \sin(\omega t + \phi)$$



or

$$V = -\omega L I_m \cos(\omega t + \underbrace{\phi - \tfrac{\pi}{2}}_{\text{phase}})$$

Thus,

$$\bar{V} = -\omega L I_m \cdot e^{j(\phi - \frac{\pi}{2})}$$

and

$$\bar{V} = -\omega L I_m e^{j\phi} \underbrace{e^{-j\frac{\pi}{2}}}_{-j} = j\omega L \underbrace{I_m e^{j\phi}}_{= \bar{I}}$$

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Thus, $\boxed{\bar{V} = j\omega L \bar{I}}$ $\Rightarrow$ $v(t) = L\dfrac{di(t)}{dt} \xrightarrow{P} \bar{V} = j\omega L \bar{I}$

$\Rightarrow \dfrac{d}{dt} \xrightarrow{P} j\omega$

3-) V-I Relation for a Capacitor:

For a capacitor,

$$i = C \cdot \frac{dv}{dt}, \quad \text{where} \quad v = V_m \cos(\omega t + \phi)$$

Then,

$$i = -C\omega V_m \sin(\omega t + \phi)$$

or $$i = -C\omega V_m \cos(\omega t + \phi - \frac{\pi}{2})$$

$$\vdots$$

$\Rightarrow \boxed{\bar{i} = j\omega C \bar{V}}$ or $\boxed{\bar{V} = \dfrac{\bar{I}}{j\omega C} = -j\dfrac{\bar{I}}{\omega C}}$ $\Rightarrow \bar{V} = Z_c \cdot \bar{I}$

Impedance & Reactance:

Define $\boxed{\bar{V} = Z\bar{I}}$ where Z is called "impedance"

$\boxed{Z = R + jX}$ is a complex number.

where R = Resistance (real part of the impedance)

X = Reactance (imag. " " " ").

Thus, for passive elements (R, L, C):

	$Z(\Omega)$	R	X
Resistor:	R	R	0
Inductor:	$j\omega L$	0	ωL
Capacitor:	$\dfrac{-j}{\omega C}$	0	$\dfrac{-1}{\omega C}$

Thus, Z: impedance is the resistance for AC circuits.

Also, define

$\boxed{Y = \text{Admittance} = \dfrac{1}{Z}}$ ($\mho$ = mho's or S: Siemens).

In addition, $Y = G + jB$

where G = Conductance

B = Susceptance.

- If $B = 0 \Rightarrow G = \dfrac{1}{R}$, if $G = 0 \Rightarrow B = \dfrac{1}{X}$

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- Kirchoff's Laws in Frequency Domain -

1-) KVL in Phasor domain:

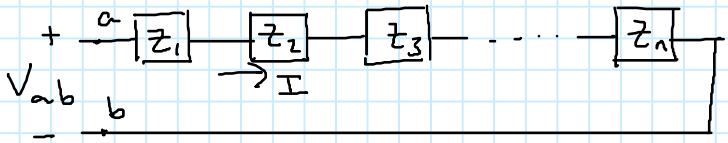
$$\bar{V}_1 + \bar{V}_2 + \cdots + \bar{V}_n = 0$$

2-) KCL in Phasor domain:

$$\bar{I}_1 + \bar{I}_2 + \cdots + \bar{I}_n = 0$$

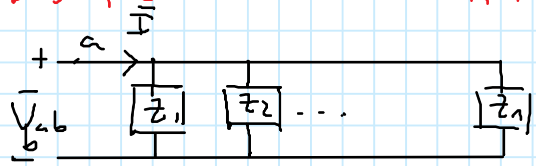
3-) Series and Parallel Connections:

a-) Series Connection



$$Z_{ab} = \frac{\bar{V}_{ab}}{\bar{I}} = Z_1 + Z_2 + \cdots + Z_n$$

b-) Parallel Connection:



$$Z_{ab} = \frac{\bar{V}_{ab}}{\bar{I}} \Rightarrow$$

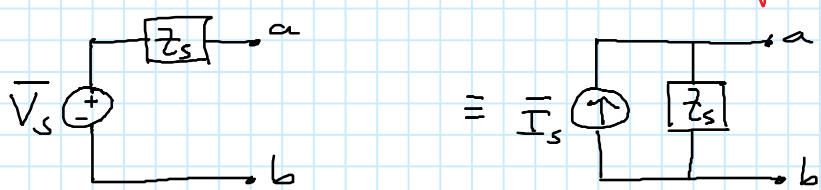
$$\frac{1}{Z_{ab}} = \frac{1}{Z_1} + \frac{1}{Z_2} + \cdots + \frac{1}{Z_n}$$

However,

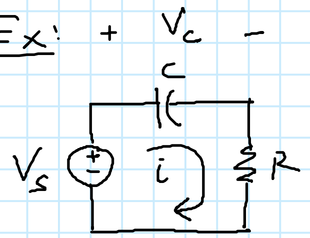
Y_{ab} = Admittance $= G + jB$

$$Y_{ab} = Y_1 + Y_2 + \cdots + Y_n$$

Source Transformation in Freq. Domain:



Ex:



$V_s = V_s(t) = V_m \cos(\omega t + \phi)$, $V_c(0) = 0$

$V_R = ?$ (steady state)

Time domain solution.

KVL equation: $-V_s + V_c + iR = 0$

or

$$V_c + iR = V_s \Rightarrow V_c + RC\frac{dV_c}{dt} = V_m \cos(\omega t + \phi)$$

$\rightarrow$ Find $V_c \Rightarrow V_R = V_s - V_c$

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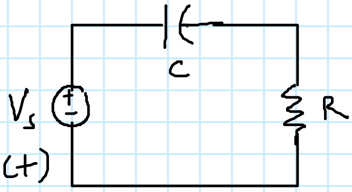
Phasor Analysis:

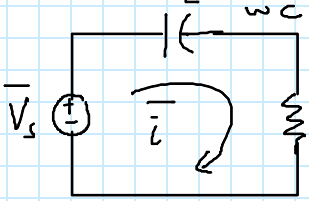


$= V_s(t)$

$= V_m \cos(\omega t + \phi)$

$\xrightarrow{P}$

$Z_c = \frac{-j}{\omega C}$



$\bar{V}_s = V_m e^{j\phi}$

$= V_m \angle \phi$

KVL:

$$\boxed{-\bar{V}_s + Z_c \bar{i} + Z_R \bar{i} = 0}$$

or

$$V_c + RC\underbrace{\frac{dv_c}{dt}}_{i(t)} = V_m \cos(\omega t + \phi) \xrightarrow{P} \boxed{\frac{-j}{\omega C} \cdot \bar{i} + R\bar{i} = V_m e^{j\phi}}$$

$$\Rightarrow \bar{i} = \frac{V_m e^{j\phi}}{R - j\frac{1}{\omega C}} \quad (A)$$

or

$$\bar{i} = \frac{V_m e^{j\phi}}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}} \cdot e^{j\tan^{-1}\left(-\frac{1}{\omega RC}\right)}}$$

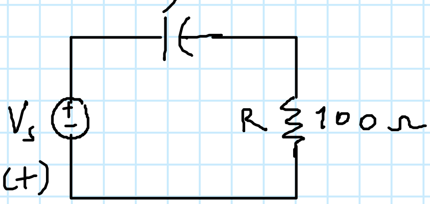
Find $\bar{i}$.

Then, $\bar{V}_R = R \cdot \bar{i}$.

Then, $V_R(t) = \mathrm{Re}[\bar{V}_R \cdot e^{j\omega t}]$.

Ex:

$C = 1\mu F$



$+$ $V_R(t) = ?$ $-$

$= V_s(t)$

$= V_m \cos(\omega t + \phi)$

$= 10 \cos(10000\pi t)$

$\xrightarrow{P}$

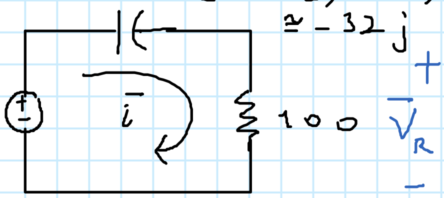
$\bar{V}_s = 10\angle 0$

$= 10 e^{j0}$

$= 10$

$\frac{-j}{\omega C} = \frac{-j}{(10000\pi)(10^{-6})}$

$\simeq -32j$



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$$\bar{i} = \frac{\bar{V}_s}{100-32j} \quad \text{(ohm's law)}$$

$$= \frac{10}{100-32j} \cong 0.1 + 0.03j$$

$$= 0.095\, e^{j\tan^{-1}\left(\frac{0.03}{0.1}\right)}$$

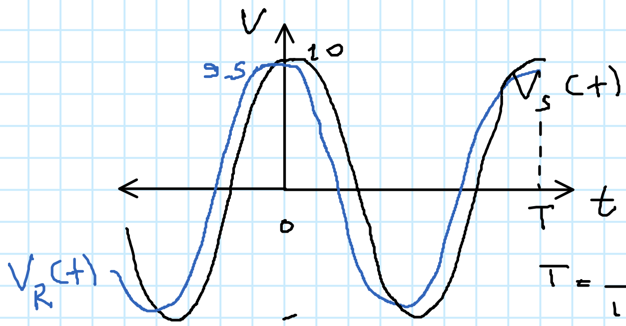
$$= 0.095\, e^{j0.3}$$

$$\bar{i} = 0.095\, e^{j17.73^\circ} \quad (A)$$

$$\Rightarrow \bar{V}_R = R.\bar{i} = 9.5\, e^{j17.73^\circ} \quad (V)$$

Time expression: $V_R(t) = Re\left[\bar{V}_R . e^{j\omega t}\right] = 9.5\, Cos(10000\pi t + 0.3)$

$\downarrow$ 17.73°



$T = \frac{1}{10000}$ sec.

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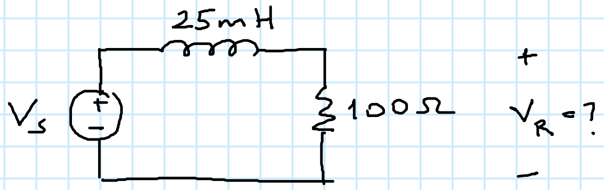
Ex:

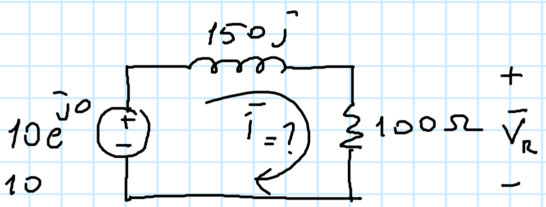


where $V_s = 10\cos(6000t)$

Find the current and the voltage across the resistor.

Ans.

Phasor circuit is:



$$\bar{I} = \frac{10}{100+150j}$$

$$= \frac{10(100-150j)}{100^2+150^2} = 0.03 - 0.046j \text{ A.}$$

$$\text{or } \bar{I} = \sqrt{(0.03)^2+(-0.046)^2}\; e^{j\tan^{-1}\left(\frac{-0.046}{0.03}\right)}$$

$$= 0.055\, e^{j(-56.8^\circ)}$$

$$\Rightarrow i(t) = \text{Re}[\bar{I}\cdot e^{j\omega t}] = \text{Re}\left[0.055 e^{j(-56.8^\circ)} \cdot e^{j6000t}\right]$$

$$= \text{Re}\left[0.055\, e^{j\underbrace{(6000t - 56.8^\circ)}_{\theta}}\right]$$

$$= 0.055\cos(6000t - 56.8^\circ)$$

$$\Rightarrow V_R(t) = i(t)\cdot R = 5.5\cos\left(6000t - 56.8^\circ \times \frac{\pi}{180}\right)$$

$$= 5.5\cos(6000t - 1)\text{ V.}$$

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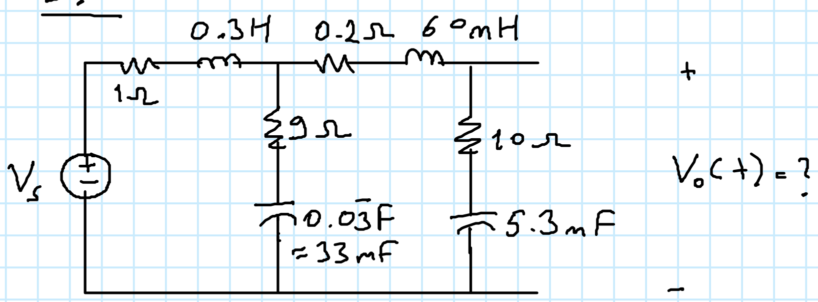
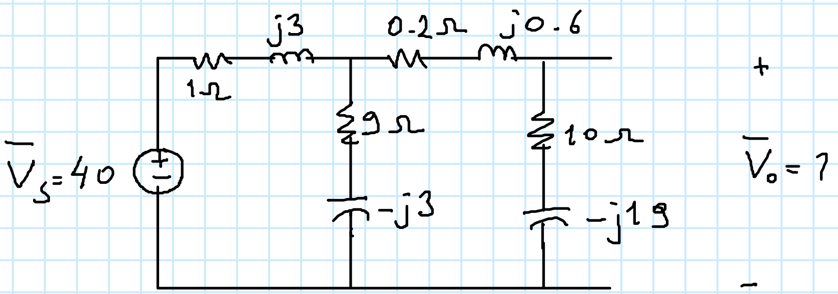
Ex:



$V_s = 40\cos(10t)$ (V)

Ans:

Phasor domain equivalent circuit is:



$$Z_{0.3H} = j\omega L = j(10)(0.3) = j3\,(\Omega)$$

$$Z_{33mF} = \frac{-j}{\omega C} = \frac{-j}{10.(0.0\bar{3})} = -j3\,(\Omega)$$

$$Z_{60mH} = j\omega L = j(10)(60\times 10^{-3}) = j0.6\,(\Omega).$$

$$Z_{5.3mF} = \frac{-j}{\omega C} = \frac{-j}{(10)(5.3\times 10^{-3})} = -j19\,(\Omega).$$

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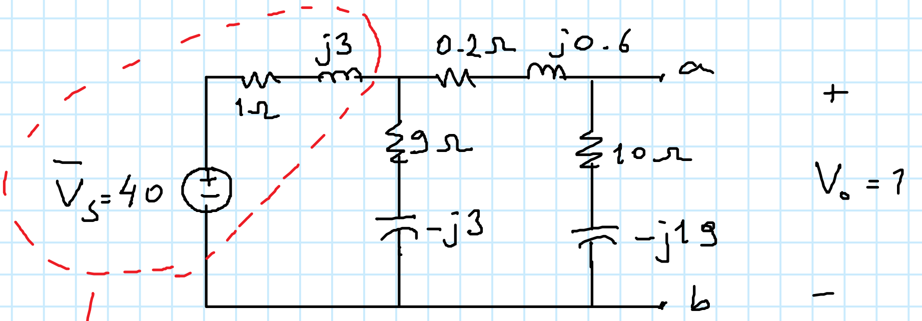
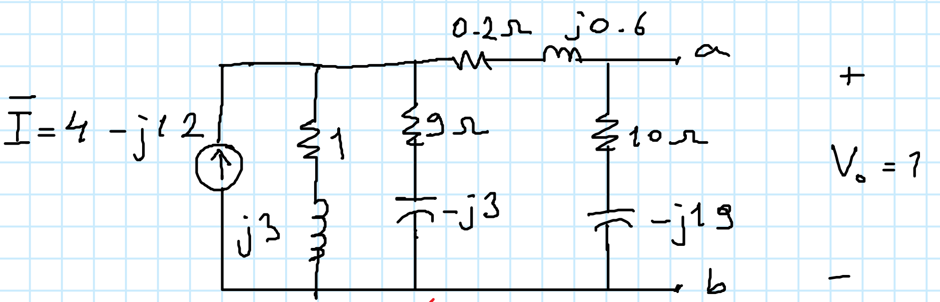


Replace by its Norton equivalent $\Rightarrow \bar{I} = \frac{40}{1+j3}\cdot\frac{(1-j3)}{(1-j3)} = \frac{40(1-j3)}{10}$

Then, we have

$\bar{I} = 4 - j12$ (A)



Parallel

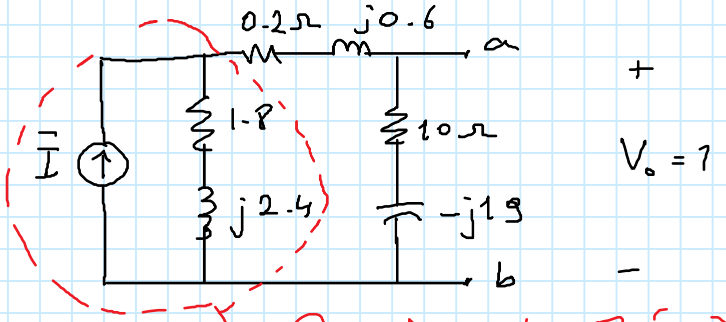
$\underbrace{}_{Z_1}$ $\underbrace{}_{Z_2}$

$$\Rightarrow Z_1 \parallel Z_2 = Z_t \Rightarrow \frac{1}{Z_t} = \frac{1}{Z_1} + \frac{1}{Z_2}$$

$$\Rightarrow Z_T = \frac{Z_1 Z_2}{Z_1 + Z_2} = \frac{(1+j3)(9-j3)}{10}$$

$$Z_T = 1.8 + j2.4 \ \Omega$$

Now, we have:



Transform to Thevenin equivalent

$$\bar{V}_{Th} = (4-j12)(1.8+j2.4) = 36 - j12 \ (V)$$

Then,

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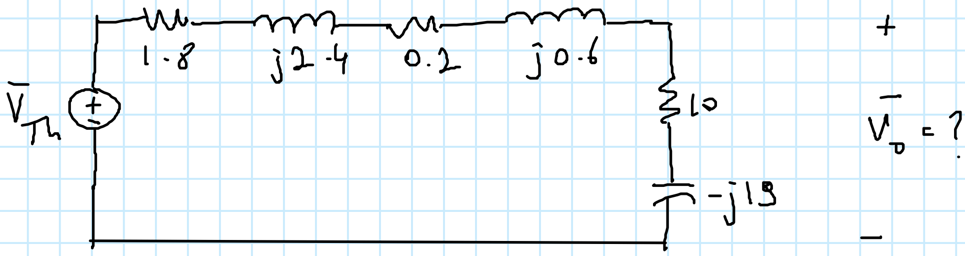

Using the voltage divider:

$$\bar{V}_o = \bar{V}_{Th} \cdot \frac{(10-j19)}{12-j16}$$

$$= (36-j12) \cdot \frac{(10-j19)}{12-j16} \cdot \frac{(12+j16)}{(12+j16)} = (36-j12) \frac{(10-j19)(12+j16)}{12^2+16^2} =$$

$$= \frac{1}{400} \cdot \underbrace{\overset{a+jb}{(36-j12)}}_{re^{j\theta}} (10-j19)(12+j16)$$

$$= \sqrt{a^2+b^2}\, e^{j\tan^{-1}\frac{b}{a}}$$

$$= \frac{1}{400} \cdot (37.95\, e^{-j18.4^\circ})(21.5\, e^{-j62.2^\circ})(20 e^{j53^\circ})$$

$$= \frac{(37.95)(21.5)(20)}{400} \cdot e^{j(53^\circ - 18.4^\circ - 62.2^\circ)}$$

$$= 40.8\, e^{-j27.6^\circ} = 40.8\, e^{j\underbrace{(-27.6)\times\frac{\pi}{180}}_{-0.48\text{ rad}}}$$

Time expression of $\bar{V}_o$ is

$$v_o(t) = Re\left[\bar{V}_o \cdot e^{j\omega t}\right] = Re\left[40.8\, \underbrace{e^{-j0.48} \cdot e^{j10t}}_{e^{j(10t-0.48)}}\right]$$

$$= Re\left[40.8\cos(10t-0.48) + j40.8\sin(10t-0.48)\right]$$

$$= 40.8\cos(10t-0.48) \quad (V)$$

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-Examples -

1-)

For the circuit below,
a) Draw the phasor domain circuit.
b) Find the current in the circuit in phasor notation and in time notation.
c) Find the voltage across the capacitor in phasor notation.

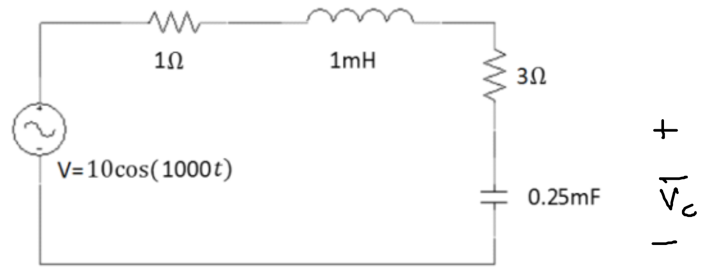
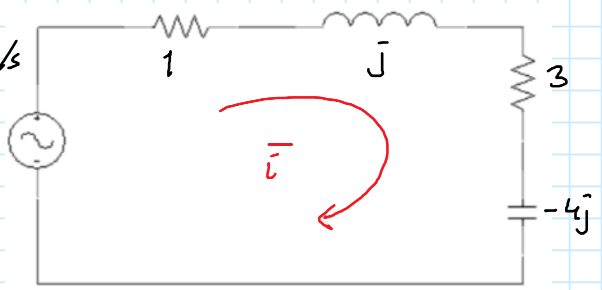


Figure 4: Circuit for question 4.

Ans:

a-) $\omega = 1000\ rad/s$

$\bar{V} = 10\, e^{j0}$
$= 10.$
$= 10\angle 0^\circ$



$Z_{1mH} = j\omega L = j(1000)(10^{-3}) = j$

$Z_{0.25mF} = \frac{-j}{\omega C} = \frac{-j}{(1000)(\frac{1}{4}\times 10^{-3})} = -4j$

b-) $$\bar{i} = \frac{\bar{V}}{Z_{total}} = \frac{10}{4-3j}\,(4+3j) = \frac{10(4+3j)}{16+9} = \frac{10}{25}(4+3j) = \frac{40}{25} + \frac{30}{25}j$$

$$= 1.6 + 1.2j \quad (A)$$

$$= \sqrt{1.6^2 + 1.2^2}\; e^{j\tan^{-1}\frac{1.2}{1.6}}$$

$$= 2e^{j36.87^\circ}$$

$$\Rightarrow i(t) = Re[\bar{i}.e^{j\omega t}] = 2\cos(1000t + 36.87^\circ \times \frac{\pi}{180})$$

$$= 2\cos(1000t + 0.6435)$$

c-) $$\bar{V}_c = \bar{V} \times \frac{(-4j)}{4-3j} = 10 \cdot \frac{-4j}{4-3j}\,(4+3j) = \frac{10}{25}(-4j)(4+3j)$$

$-12j^2 = 12$

$$= \frac{10}{25}(-16j + 12) = \frac{120}{25} - \frac{160}{25}j = 4.8 - 6.4j \quad (V).$$

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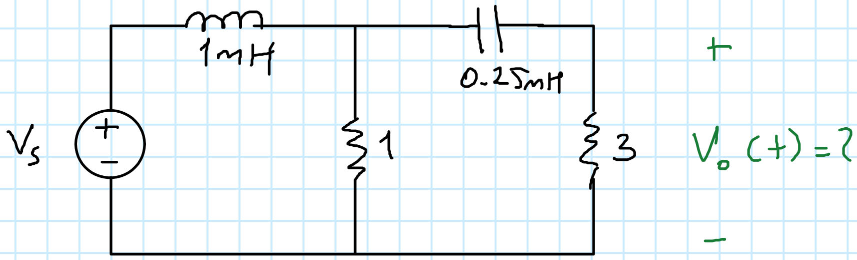


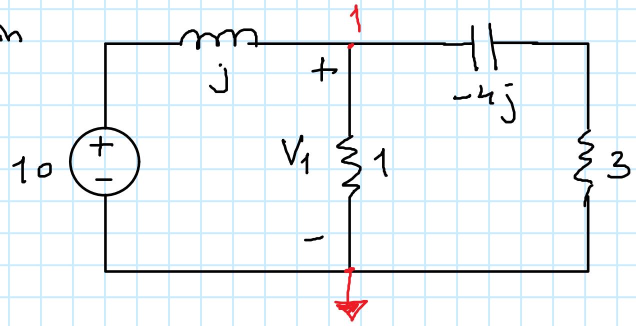
$V_s = 10\cos(1000t)$ V.

Find $V_o(t)$. $\omega = 2\pi f \Rightarrow f = \frac{1000}{2\pi} \simeq 160$ Hz

Ans:

Phasor domain circuit:

$\omega = 1000$



Using the Node-Voltage method.

$$\frac{\bar{V}_1 - 10}{j} + \frac{\bar{V}_1}{1} + \frac{\bar{V}_1}{3-4j} = 0$$

$(3-4j) \quad j(3-4j) \quad (j)$

$$(3-4j)(\bar{V}_1 - 10) + \bar{V}_1\, j(3-4j) + \bar{V}_1\, j = 0$$

$$\cancel{3\bar{V}_1} - 30 - \cancel{4j\bar{V}_1} + 40j + \cancel{3j\bar{V}_1} + \cancel{4\bar{V}_1} + \cancel{\bar{V}_1 j} = 0$$

$$7\bar{V}_1 - 30 + 40j = 0$$

or $\bar{V}_1 = \frac{1}{7}(30 - 40j) = 4.3 - 5.7j$ (V).

From Voltage division:

$$\bar{V}_o = \bar{V}_1 \cdot \frac{3}{3-4j} = (4.3 - 5.7j)\frac{(3)}{3-4j} = \frac{3(3+4j)(4.3-5.7j)}{25}$$

$(3+4j)$

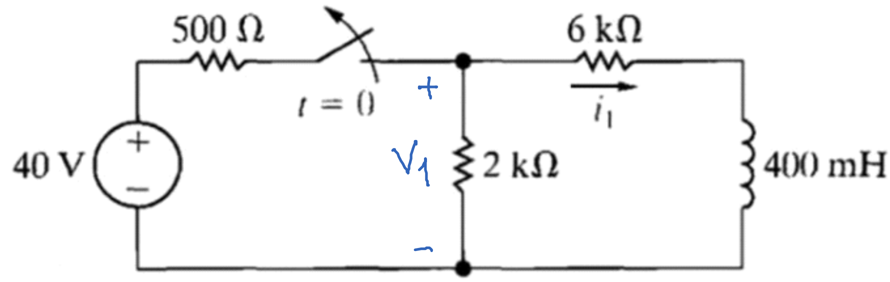
$$= \frac{3}{25} \cdot (12.9 - 17.\cancel{1}j + 17.\cancel{2}j + 22.8)$$

(approx.)

$$= \frac{3}{25}(35.7) = 4.28\,V.$$

$\Rightarrow V_o(t) = 4.28\cos(1000t)$

3-) For the circuit given below



Find $i_1(t)$ for $t \geq 0$. (20 points)

Ans: Variable of interest = Inductor current.

Initial values: $i_1(0^-) = ?$

For $t < 0$:

$$V_1 = (40V)\frac{(2k \| 6k)}{500 + (2k \| 6k)} = \cancel{40}^{10} \cdot \frac{\cancel{1500}^{3}}{\cancel{2000}_{4}} = 30V.$$

$$\Rightarrow i_1(0^-) = \frac{30}{6k} = 5\,mA. = i_1(0^+)$$

$i_1(\infty) = 0$ final value.

$$\tau = \frac{L}{R} = \frac{400 \times 10^{-3}}{8000} = \frac{\cancel{4} \times 10^{-3}}{{}_2\cancel{8}^{1} \times 10^{1}} = 0.5 \times 10^{-4} = 50\,\mu sec.$$

$$i_1(t) = i_{1f} + [i_1(0) - i_{1f}]e^{-(t-t_0)/\tau}$$

or $$i_1(t) = 5e^{-t/\tau}\,mA. = 5e^{-\frac{t}{\frac{1}{2}\times 10^{-4}}} = \boxed{5e^{-20000t}\,mA.}$$

4-)

For $t < 0$, the current is already settled. Initial charge of the capacitor is 0V. For the circuit below, find
a) i(t) for $t \geq 0$,
b) v(t) for $t \geq 0$.

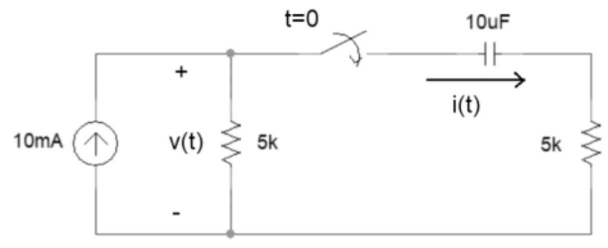
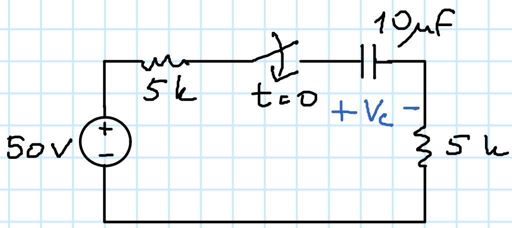


Figure 1: Circuit for question 1.

<u>Ans</u> This is the step response of an RC-circuit.
The equivalent circuit is:

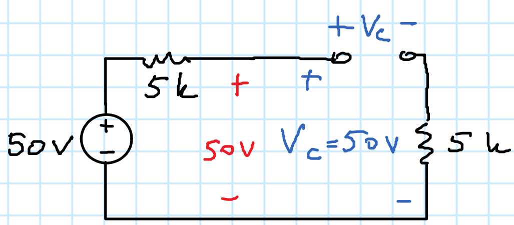


Variable of interest is the capacitor voltage (standart)

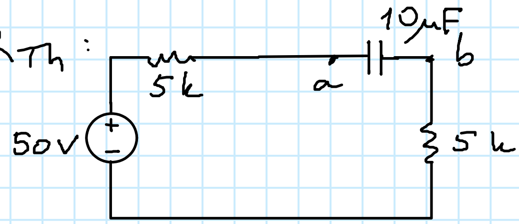
$V_c(0^-) = 0 = V_c(0^+) = V_c(0)$

$V_c(\infty) = 50V$.

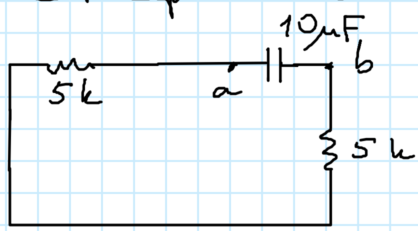
$\tau = RC$



R_{Th}:



De-activate the indep sources



$\Rightarrow R_{Th} = 10k$.

$\Rightarrow \tau = (10k)(10\mu F) = 10^5 \times 10^{-6} = 0.1$ sec

$V_c(t) = V_c(\infty) + [V_c(0) - V_c(\infty)]e^{-(t-0)/0.1}$

$= 50 - 50e^{-10t}$ (V)

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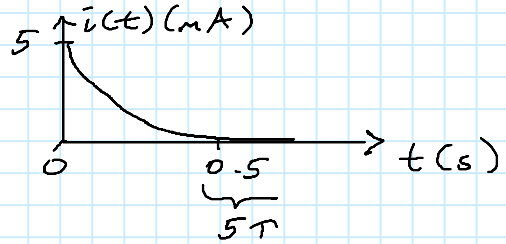
Thus,

$$i(t) = C\,\frac{dV_c(t)}{dt} = (10\times10^{-6}) \times \frac{d}{dt}\left(50 - 50e^{-10t}\right)$$

$$= (10^{-5}) \times 500\,e^{-10t}$$

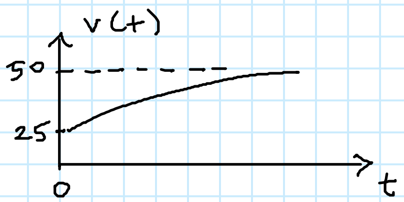
$$= 5\times10^{-3}\,e^{-10t} = 5e^{-10t}\ mA$$



b-) $v(t) = (5k)\left[10mA - i(t)\right]$

$$= 5\cancel{000}\left(10 - 5e^{-10t}\right]\cancel{m}A$$

$$= 50 - 25e^{-10t}\ V.$$



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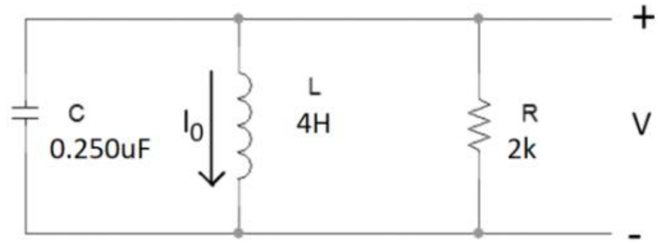
5-)

For the circuit given below, it is given that V_0=0V, I_0=-30mA.



a-) Determine the type of the response. Show your work.

b-) Find $v(t)$, $t \geq 0$.

Ans: This is a parallel RLC circuit natural response.

a-) $\alpha = \frac{1}{2RC} = \frac{1}{\cancel{2}(\cancel{2}0\cancel{0}0)(\frac{1}{\cancel{4}} \times 10^{-\cancel{6}})^{-3}} = 1000$

$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\cancel{4} \times \frac{1}{\cancel{4}} \times 10^{-6}}} = \frac{1}{10^{-3}} = 1000$

$\alpha^2 = \omega_0^2 \Rightarrow$ critically damped!

b-) $S_1 = S_2 = -\alpha = -1000$

The response is $v(t) = D_1 t e^{-\alpha t} + D_2 e^{-\alpha t}$

I.C's : $v(0^+) = D_2 = 0$. , $\frac{dv(0^+)}{dt} = \frac{i_C(0^+)}{C} = D_1 - \alpha \cancel{D_2}^{\,0}$

$\Rightarrow D_1 = \frac{1}{C} i_C(0^+) = \frac{1}{C}[-i_0(0^+)]$

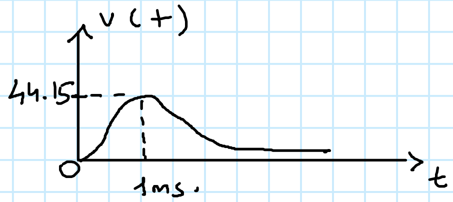
$= \frac{1}{\frac{1}{4} \times 10^{-6}} \cdot (30mA)$

$= \frac{4 \times 30 \times 10^{-\cancel{3}}}{10^{-\cancel{6}}}$

$= 4 \times 30 \times 10^3$

$= 120000$

$\Rightarrow v(t) = 120000 t e^{-1000t}$ V.



$\frac{dv(t)}{dt} = 0$ gives $\cancel{120000} \cancel{e^{-1000t}} = \cancel{120000}t(1000)\cancel{e^{-1000t}}$

$t = 1$ ms.

$v(1ms) = 120000(10^{-3})e^{-1} = 120 \times \frac{1}{e} = \frac{120}{2.718\ldots} = 44.15$

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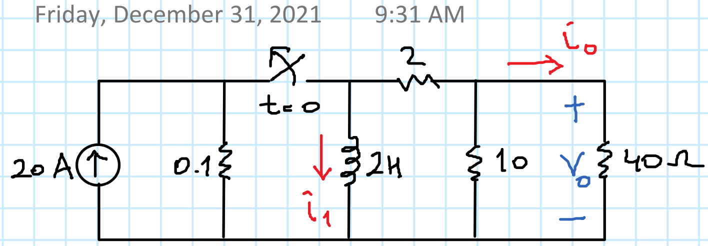
6-)



Find

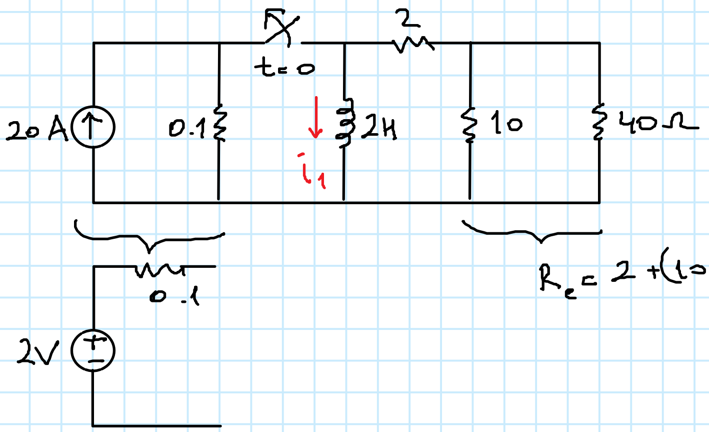
a-) $i_1(t)$ for $t \geq 0$

b-) $i_o(t)$ for $t \geq 0$

c-) $V_o(t)$ for $t \geq 0$

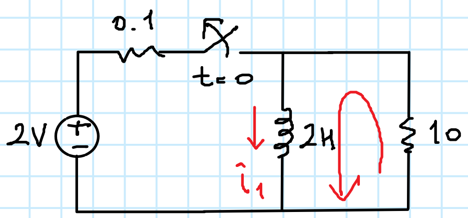
Ans.

a-) The variable of interest is the inductor current



$R_e = 2 + (10 \parallel 40) = 2 + 8 = 10\Omega$

Then, we have



This is the natural response of an RL-circuit.

The general solution is

$$i_1(t) = i_{1f} + [i_1(0) - i_{1f}]e^{-t/\tau}$$

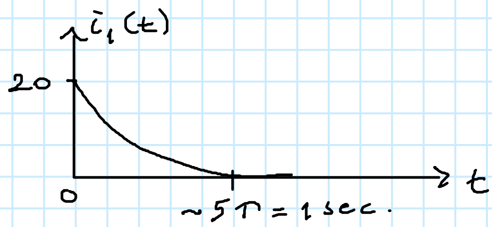
$i_1(0^-) = i_1(0^+) = i_1(0) = \frac{2}{0.1} = 20\ A$

$i_{1f} = i_1(\infty) = 0$

$\tau = \frac{L}{R} = \frac{2}{10} = 0.2$

Substitute the values into the formula

$$\Rightarrow \quad i_1(t) = 20\, e^{-\frac{t}{0.2}} = 20e^{-5t} \quad (A)$$



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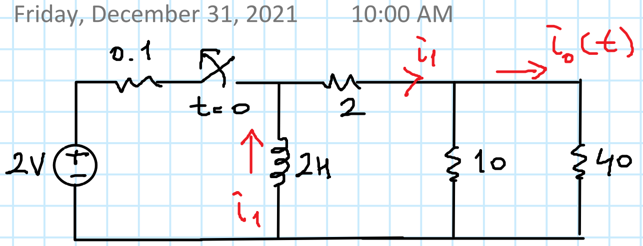
b-)



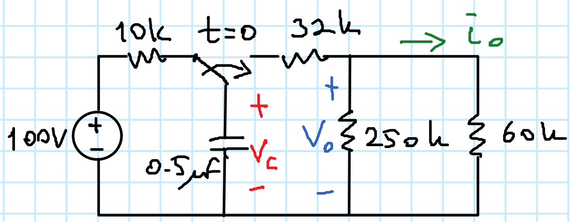
$i_o(t) = ?$, $t \geq 0$

Using the current division

$$i_o(t) = i_1(t) \cdot \frac{10}{50} = \frac{1}{5} i_1(t) = -4e^{-5t}, \quad t \geq 0$$

c-) $V_o(t) = 40\, i_o(t) = -160e^{-5t}$, $t \geq 0$

7-)



Find

a-) $V_c(t)$, $t \geq 0$, d-) $E_{60k\Omega} = ?$

b-) $V_o(t)$, $t \geq 0$

c-) $i_o(t)$, $t \geq 0$

Ans:

a-) The variable of interest is the capacitor voltage $V_c(t)$

$$V_c(t) = V_{cf} + [V_c(0) - V_{cf}]e^{-t/\tau}$$

where

$V_c(0) = V_c(0^-) = 100\,V.$

$V_{cf} = 0\,V.$

$\tau = RC \Rightarrow R_{Th} = 32k + (260k \parallel 60k) = 80k$

Then, $\tau = (80k)(\frac{1}{2})(10^{-6}) = (8)(10^4)(\frac{1}{2})(10^{-6}) = 4 \times 10^{-2} = 0.04$

Thus,

$$V_c(t) = 100e^{-t\left(\frac{1}{4\times10^{-2}}\right)} = 100e^{-t(0.25\times100)} = 100e^{-25t}\ V.$$

b-) $V_o(t) = V_c(t) \dfrac{(250k \parallel 60k)}{(250k \parallel 60k) + 32k} = V_c(t) \cdot \dfrac{\cancel{48k}^{\,3}}{\cancel{80k}_{\,5}} = \dfrac{3}{5} V_c(t) = 60e^{-25t}\ V$

c-) $i_o(t) = \dfrac{V_o(t)}{60k} = e^{-25t}\ mA$

d-) $P_{60k} = i_o^2(t)\, R = (e^{-50t} \times 10^{-6})(60k) = 6\times10^{-2} e^{-50t} = 0.06e^{-50t}\ W$

$= 60e^{-50t}\ mW.$

$$E_{60k} = \int_0^\infty P_{60k}\, dt = \int_0^\infty 60e^{-50t} dt = 60\int_0^\infty e^{-50t} dt = 60\left(\frac{-1}{50}e^{-50t}\Big|_0^\infty\right)$$

$$= \frac{-6}{5}\left(\cancel{e^{-\infty}}^{\,0} - e^0\right) = \frac{6}{5}\,J. = 1.2\,J.$$

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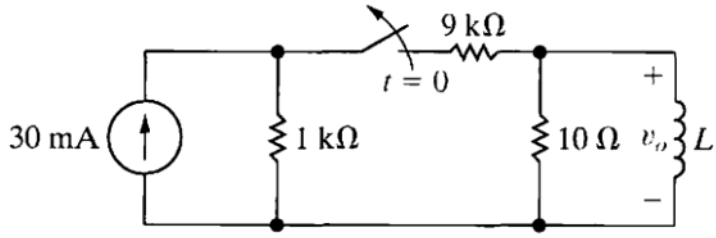
8-) In the circuit in Fig. P7.10, the switch has been closed for a long time before opening at $t = 0$.

a) Find the value of L so that $v_o(t)$ equals $0.5\, v_o(0^+)$ when $t = 1$ ms.

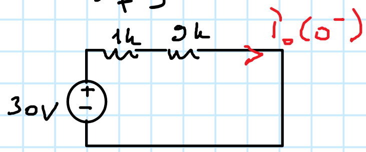


Ans:

This is the natural response of an RL-circuit

The variable of interest is the inductor voltage

$$i_o(t) = i_{of} + [i_o(0) - i_{of}]e^{-t/\tau} \quad , i_o(t) = \text{Inductor current}$$



$$i_o(0) = i_o(0^+) = i_o(0^-) = ?$$

$$= \frac{30}{10k} = 3\,mA.$$

$$i_o(\infty) = 0$$

$$\tau = \frac{L}{R} = \frac{L}{10}$$

Thus,

$$i_o(t) = 3e^{-\frac{10t}{L}}\ mA$$

Then, $V_o(t) = L\frac{di_o(t)}{dt} = L\cdot\frac{d}{dt}(3e^{-\frac{10}{L}t}\times 10^{-3}) = L(3\times10^{-3})\cdot\left(\frac{-10}{L}e^{-\frac{10}{L}t}\right)$

Given that $V_o(1ms) = 0.5\,V_o(0^+)$

$$\Rightarrow \cancel{L}(3\times\cancel{10^{-3}})\left[\frac{-\cancel{10}}{\cancel{L}}e^{-\frac{10}{L}(10^{-3})}\right] = \frac{1}{2}\cancel{L}(3\times\cancel{10^{-3}})\left(\frac{-\cancel{10}}{\cancel{L}}\right)$$

$$e^{-\frac{10^{-2}}{L}} = \frac{1}{2} \quad \Rightarrow e^{\frac{10^{-2}}{L}} = 2$$

$$\frac{-10^{-2}}{L} = \ln\frac{1}{2}$$

$$\frac{10^{-2}}{L} = \ln 2$$

$$L = \frac{10^{-2}}{0.69} = \frac{0.01}{0.7}$$

$$L = \frac{10^{-2}}{\ln 2} = \frac{0.01}{\ln 2}$$

$$= 14.28\,mH.$$

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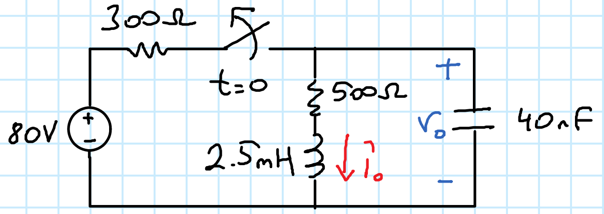
9-)



The swith in the circuit has been closed for a long time. The switch opens at $t=0$

Find a-) $i_o(t)$, $t \geqslant 0$

b-) $V_o(t)$, $t \geqslant 0$.

Ans.

a-) For $t \geqslant 0$, this is a series RLC circuit natural response

Thus, for $t<0$

$$i_o(0) = i_o(0^+) = i_o(0^-) = \frac{80}{500+300} = \frac{\cancel{80}^{\,1}}{\cancel{800}_{\,10}} = 0.1\ A = 100\ mA$$

Also,

$$V_o(0) = V_o(0^+) = V_o(0^-) = 80 \cdot \frac{500}{500+300} = \cancel{80}^{\,10} \cdot \frac{5}{\cancel{8}} = 50\ V$$

Now that we have the initial conditions, we need to find the type of the response: (Look at pg-63)

$$\alpha = \frac{R}{2L}, \quad \omega_o = \frac{1}{\sqrt{LC}}$$

or

$$\alpha = \frac{500}{2(2.5\times10^{-3})} = 10^5\ rad/s$$

$$\omega_o^2 = \frac{1}{LC} = \frac{1}{(2.5\times10^{-3})(40\times10^{-9})} = 100\times10^8 = 10^{10}$$

$$\Rightarrow \omega_o = 10^5$$

Then, $\alpha = \omega \Rightarrow$ Critically damped!

$$i(t) = D_1 t e^{-\alpha t} + D_2 e^{-\alpha t}$$
$$= D_1 t e^{-10^5 t} + D_2 e^{-10^5 t}$$

$$\Rightarrow i_o(0) = D_2 = 100\ mA$$

$$\frac{di_o(0)}{dt} = \frac{V_L(0^+)}{L} = -\alpha D_2 + D_1 = 0 \Rightarrow D_1 = 10^5(100\times10^{-3}) = 10000.$$

Thus,

$$i_o(t) = 10000\, t e^{-10^5 t} + 0.1 e^{-10^5 t}\ A$$

b-) $v_o(t) = D_3 t e^{-10^5 t} + D_4 e^{-10^5 t}\ V.$

$$v_o(0) = D_4 = 50, \quad C\frac{dv_o(0)}{dt} = -0.1$$

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$$\frac{dV_o(0)}{dt} = \frac{-0.1}{40\times10^{-9}} = -25\times10^{5}\ \frac{V}{s} = -\alpha D_4 + D_3$$

$$\Rightarrow D_3 = 10^5(50) - 25\times10^5 = 25\times10^5$$

$$\Rightarrow V_o(t) = 25\times10^5\, t e^{-10^5 t} + 50 e^{-10^5 t}\ V, \quad t \geq 0.$$