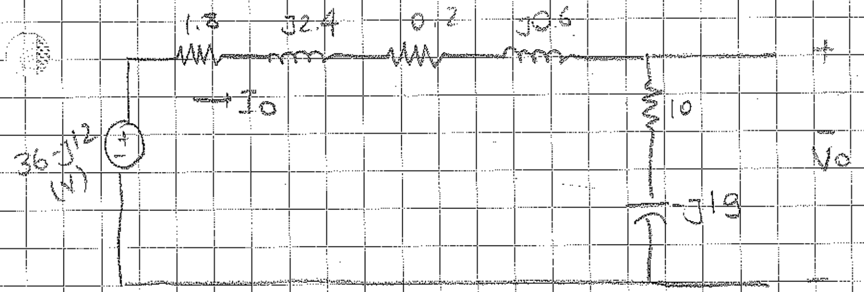
Thus,



$$I_0 = \frac{36-j12}{12-j16} = \frac{12(3-j1)}{4(3-j4)} = \frac{39+j27}{25} = \boxed{1.56+j1.08\ A}$$

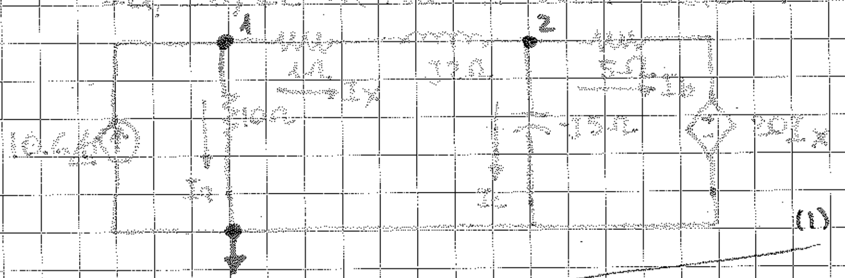
and $V_0 = (1.56+j1.08)(10-j19)$

$$\boxed{V_0 = 26.12+j18.84\ V}$$

16/11/2015

• The same concepts apply when we use the node-voltage method and the mesh current method to analyze the frequency domain circuits.

× Ex: Use the node-voltage method to find the branch currents I_a, I_b, I_c in the circuit below.



Ans: KCL at node-1

$$-10.6 + \frac{V_1}{10} + \frac{V_1 - V_2}{1+j2} = 0$$

(1) $V_1(1.1 + j0.2) - V_2 = 10.6 + j21.2$

KCL at node-2

(2) $$\frac{V_2 - V_1}{1+j2} + \frac{V_2}{-j5} + \frac{V_2 - 20I_x}{5} = 0$$

The controlling current I_x is:

(3) $$I_x = \frac{V_1 - V_2}{1+j2}$$

Substitute (3) in (2) and multiply by $(1+j2)$ given

(4) $-5V_1 + (4.8 + j0.6)V_2 = 0$

From (1) and (4)

$V_1 = 68.4 - j16.8\ V$

$V_2 = 68 - j26\ V$

Thus the branch currents are

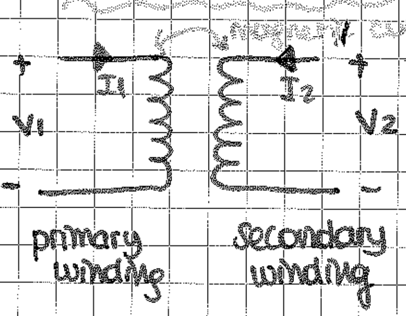
$$I_a = \frac{V_1}{10} = 6.84 - j1.68\ A$$

$$I_x = \frac{V_1 - V_2}{1+j2} = 3.76 + j1.68\ A$$

$$I_b = \frac{V_2 - 20I_x}{5} = -1.44 - j11.92\ A$$

$$I_c = \frac{V_2}{-j5} = 5.2 + j13.6\ A$$

TRANSFORMERS

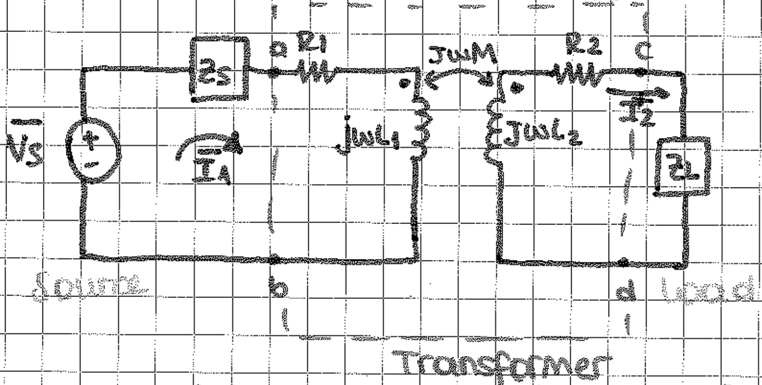


$\boxed{V_1, I_1 \;\alpha\; V_2, I_2 \text{ (coupling)}}$

→ Transformers are used in electronics, communication and power circuits for different purposes.

- The Analysis of a Linear Transformer Circuit -

Consider the circuit below:



where;

R_1 : Resistance of the primary winding
R_2 : Resistance of the secondary winding
L_1 : Self inductance of the 1° winding
L_2 : Self inductance of the 2° winding
M : Mutual inductance

→ We want to find I_1 and I_2 as a function of the above parameters

The mesh current equations are:

$$\overline{V_s} = (Z_s + R_1 + (j\omega L_1))\overline{I_1} - j\omega M\overline{I_2}$$

$$0 = -j\omega M\overline{I_1} + (R_2 + j\omega L_2 + Z_L)\overline{I_2}$$

Let us define : $Z_{11} = Z_s + R_1 + j\omega L_1$ and $Z_{22} = R_2 + j\omega L_2 + Z_L$

Then, $\overline{I_1} = \dfrac{Z_{22}}{Z_{11}Z_{22} + \omega^2 M^2}\overline{V_s}$, $\overline{I_2} = \dfrac{j\omega M}{Z_{11}Z_{22} + \omega^2 M^2} = \dfrac{j\omega M}{Z_{22}}\overline{I_1}$

Also, $\dfrac{\overline{V_s}}{\overline{I_1}} = Z_{int} = \dfrac{Z_{11}Z_{22} + \omega^2 M^2}{Z_{22}} = Z_{11} + \dfrac{\omega^2 M^2}{Z_{22}}$ and

$$Z_{ab} = Z_{int} - Z_s = Z_{11} + \frac{\omega^2 M^2}{Z_{22}} - Z_s$$

$$= R_1 + j\omega L_1 + \underbrace{\frac{\omega^2 M^2}{j\omega L_2 + R_2 + Z_L}}_{Z_r \text{ : Inflected impedence}}$$

* Z_r depends on the coupling M, Thus, it is called reflected impedence from the 2nd circuit)

* The load impedence is : $Z_L = R_L + jX_L$ then, the reflected impedence expression becomes :

$$Z_r = \frac{\omega^2 m^2}{R_2 + R_L + j(\omega L_2 + X_L)} = \frac{\omega^2 m^2 [(R_2+R_L) - j(\omega L_2 + X_L)]}{(R_2+R_L)^2 + (\omega L_2 + X_L)^2}$$

$$= \frac{\omega^2 m^2}{|Z_{22}|^2}\left[(R_2+R_L) - j(\omega L_2 + X_L)\right]$$

where ; $Z_{22} = R_2 + R_L + j(\omega L_2 + X_L)$

$|Z| = \sqrt{X^2+R^2}$, $\tan^{-1}(X/R)$

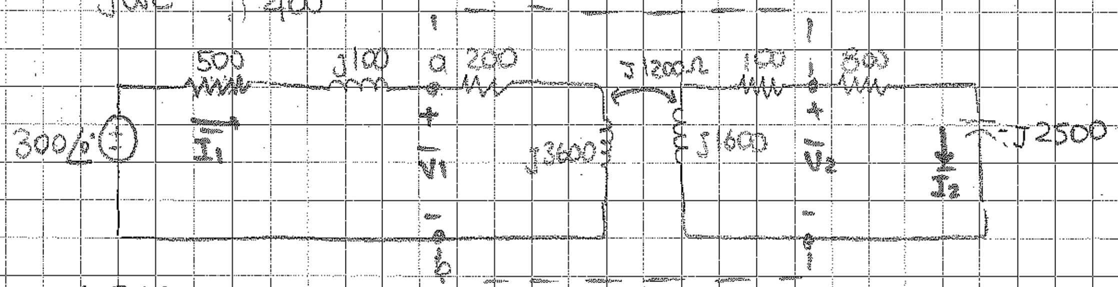
Ex : The parameters of a linear transformer are

$R_1 = 200\,\Omega$, $R_2 = 100\,\Omega$, $L_1 = 9\,\Omega$, $L_2 = 4H$ and $k = 0.5$

The transformer couples an impedence consisty of $800\,\Omega$ in series with $1\,\mu F$ capacitor. 300 V rms source has an internal impedence of $500 + j100\,\Omega$ and frequency of 400 rad/sec

a) Freq. domain circuit = ? b) Self impedence of primary circuit = ?
c) Self imp of 2° circuit = ? d) Impedence reflected into the 1° winding = ?
e) Scaling factor for reflected imp = ? f) Imp. seen looking into the 1° terminals of the transformer = ?
g) Thevenin equivalent wrt terminals c, d = ?

a) $j\omega L_1 = j(400)(9) = j3600\,\Omega$ $\quad m = 0.5\sqrt{(9)(4)} = 3H$
$j\omega L_2 = j(400)(4) = j1600\,\Omega$ $\quad j\omega m = j(400)(3) = j1200\,\Omega$
$\frac{1}{j\omega C} = \frac{10^6}{j400} = -j2500\,\Omega$ (C)



b) Self impedence of primary circuit = $Z_{11} = 500 + j100 + 200 + j3600 = 700 + j3700\,\Omega$

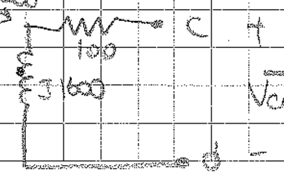
c) $Z_{22} = j1600 + 100 + 800 - j2500 = 900 - j900\,\Omega$

d) $Z_r = \left(\frac{1200}{|900-j900|}\right)^2 (900 + j900) = \frac{8}{9}(900 + j900) = 800 + j800\,\Omega$

e) The scaling factor : $\frac{Z_r}{Z_{22}} = \frac{8}{9}$

f) $Z_{ab} = 200 + j3600 + 800 + j800 = 1000 + j4400\,\Omega$

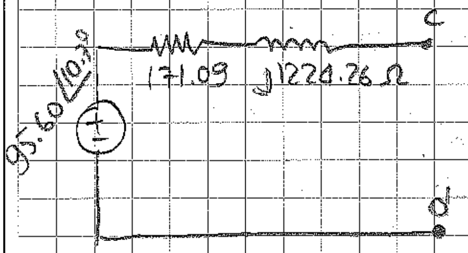
g) $\bar{V}_{cd} = (j1200)\bar{I}_1$ (open circuit)



$\bar{I}_1 = \frac{300\angle 0^\circ}{700 + j3700} = 79.67 \angle -79.23^\circ$ mA

$V_{TH} = (j1200)(79.67 \angle -79.23^\circ) = 95.60 \angle 10.71^\circ$

$$Z_{TH} = \underbrace{100 + j1600}_{\text{Impedence of the 2° winding}} + \underbrace{\left(\frac{1200}{|700 + j3700|}\right)^2 (700 - j3700)}_{Z_r \text{ from 1° winding}} = 171.09 + j124.26\,\Omega$$



→ Thevenin equivalent w.r.t c,d terminals

18/12/2015

IDEAL TRANSFORMER

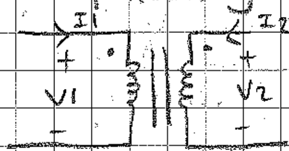
It has the following properties:

1) The coefficient of coupling $k = 1$

2) Self inductance $L_1 = L_2 = \infty$

3) Coil losses are negligible

The following symbol is used for an ideal transformer:



where we have the following relations:

$\boxed{\frac{V_1}{N_1} = \frac{V_2}{N_2}}$ and $\boxed{I_1 N_1 = I_2 N_2}$ where N_1 and N_2 are number of turns for the 1° and the 2° windings respectively

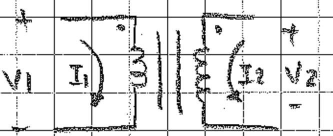
- Determining the Polarity of the Voltage & Current Ratios

→ If the coil voltages V_1 and V_2 are both (+) or (-) at the dot terminal use a (+) sign in equation:

$\left|\frac{V_1}{N_1}\right| = \left|\frac{V_2}{N_2}\right|$ Otherwise, use a (-) sign

→ If the coil currents I_1 and I_2 are both directed into or out of the dot terminal, use a (-) sign in the eqn $|I_1 N_1| = |I_2 N_2|$. Otherwise, use the (+) sign

Thus, we have the following cases:

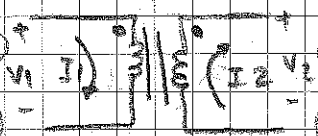


$\frac{V_1}{N_1} = \frac{V_2}{N_2}$

$N_1 I_1 = -N_2 I_2$

$\frac{V_1}{N_1} = -\frac{V_2}{N_2}$

$N_1 I_1 = N_2 I_2$



$\frac{V_1}{N_1} = \frac{V_2}{N_2}$

$N_1 I_1 = N_2 I_2$

$\frac{V_1}{N_1} = -\frac{V_2}{N_2}$

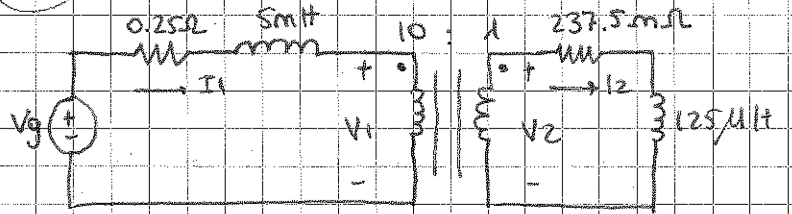
$N_1 I_1 = -N_2 I_2$

→ There are three ways to show turn ratios:

We use "a" to donate the ratio:

$$\boxed{a = \frac{N_2}{N_1}}$$

Ex: For the given circuit:



$V_g = 2500\cos(400t)$

Find the steady-state expressions for
a) $i = ?$ b) $V_1 = ?$ c) $i_2 = ?$ d) $V_2 = ?$

Ans: $\bar{V_g} = 2500\angle 0°$

$P\{5mH\} = j2\,\Omega$

$P\{125\mu H\} = j0.05\,\Omega$

a) KVL in primary circuit given:

$2500\angle 0° = (0.25 + j2)\bar{I_1} + \bar{V_1}$

$\bar{V_1} = 10\bar{V_2} = 10\left[(0.2375 + j0.05)\bar{I_2}\right]$

Because $\bar{I_2} = 10\bar{I_1}$ we have $\bar{V_1} = 10\left[(0.2375 + j0.05)10\bar{I_1}\right]$

$\bar{V_1} = (23.75 + j0.05)\bar{I_1}$

then, $2500\angle 0° = (25 + j2)\bar{I_1}$ or $\boxed{\bar{I_1} = 100\angle -16.26°\ A}$

Thus $\boxed{i_1 = 100\cos(400t - 16.26°)\ A}$

b) $V_1 = 2500\angle 0° - \left[(100\angle -16.26°)(0.25 + j2)\right] = 2500 - 80 - j185$

$V_1 = 2420 - j185 = 2427.06\angle -4.37°\ V$

Hence, $\boxed{V_1 = 2427.06\cos(400t - 4.37°)\ V}$

c) $\bar{I_2} = 10\bar{I_1} = 1000\angle -16.26°\ A$

then $\boxed{I_2 = 1000\cos(400t - 16.26°)\ A}$

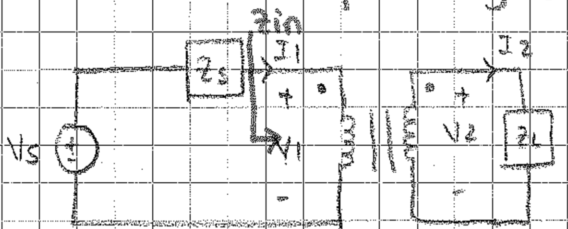
d) $\bar{V_2} = \frac{1}{10}\bar{V_1} = 242.71\angle -4.37°\ V$

then $\boxed{V_2 = 242.71\cos(400t - 4.37°)\ V}$

lag = gerisinde kalmak
lead = ilerisinde kalmak

THE USE OF AN IDEAL TRANSFORMER FOR IMPEDENCE MATCHING

Consider the following circuit



$\bar{V_1} = \dfrac{\bar{V_2}}{a}$ and $\bar{I_1} = a \cdot \bar{I_2}$

Thus the impedence seen by the source is:

$$Z_{in} = \frac{\bar{V_1}}{\bar{I_1}} = \frac{1}{a^2}\frac{\bar{V_2}}{\bar{I_2}}$$

Since, $\dfrac{\bar{V_2}}{\bar{I_2}} = Z_L$, $\boxed{Z_{in} = \dfrac{1}{a^2} Z_L}$

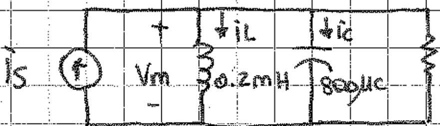
Thus, the secondary coil reflects the load impedence back to primary coil by $1/a^2$.

We use ideal transformer to match Z_s and Z_L, but this is the subject of the next chapter

• Phasor Diagrams

Steady state sinusoidal circuit analysis can be shown in the complex phasor diagram or graph.

× **EX**: For the following circuit:



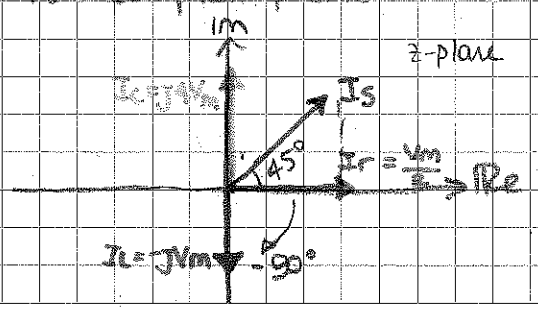
Use the phasor diagram to find the value of R that will cause i_R to lag the source current i_s by 45° when $w = 5$ krad/sec

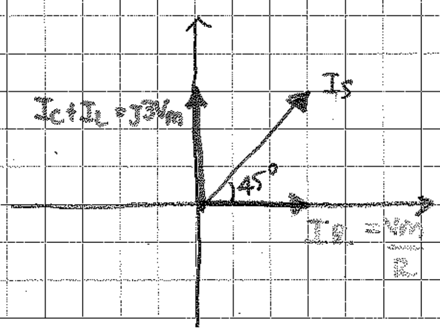
Ans: $\bar{I_L} = \dfrac{\bar{V}_m \angle 0°}{j(5000)(0.2\times10^{-3})} = V_m \angle -90°$

$\bar{I_C} = \dfrac{V_m \angle 0°}{-j/(5000)(800\times10^{-6})} = 4 V_m \angle 90°$

$I_R = \dfrac{V_m \angle 0°}{R} = \dfrac{V_m}{R} \angle 0°$

The complex plane:





$\dfrac{V_m}{R} = |I_C + I_L| = 3 V_m$

$\boxed{R = \dfrac{1}{3}\,\Omega}$