THE CRITICALLY DAMPED VOLTAGE RESPONSE

$$S_1 = S_2 = -\alpha = \frac{-1}{2RC}$$

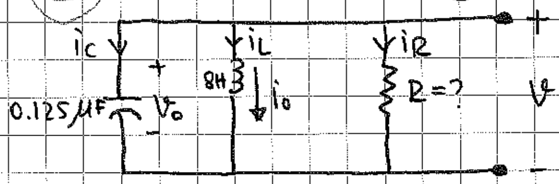
The response is:

$$\boxed{v(t) = D_1 t e^{-\alpha t} + D_2 e^{-\alpha t}}$$

I.C's:

$$\frac{dv(0^+)}{dt} = \frac{i_C(0^+)}{C} = D_1 - \alpha D_2$$

EX: For the circuit given below:



$V_0 = 0$, $I_0 = -12.25\,mA$

a) Find the value for R such that the response is critically damped

b) Find $v(t)$.

Ans: a) $\omega_0 = \frac{1}{\sqrt{LC}} = \sqrt{\frac{10^6}{8(0.125)}} = 10^3$ rad/s

In critically damped response $\alpha^2 = \omega_0^2$, $\alpha = \omega_0$

$\alpha = 10^3$ rad/s

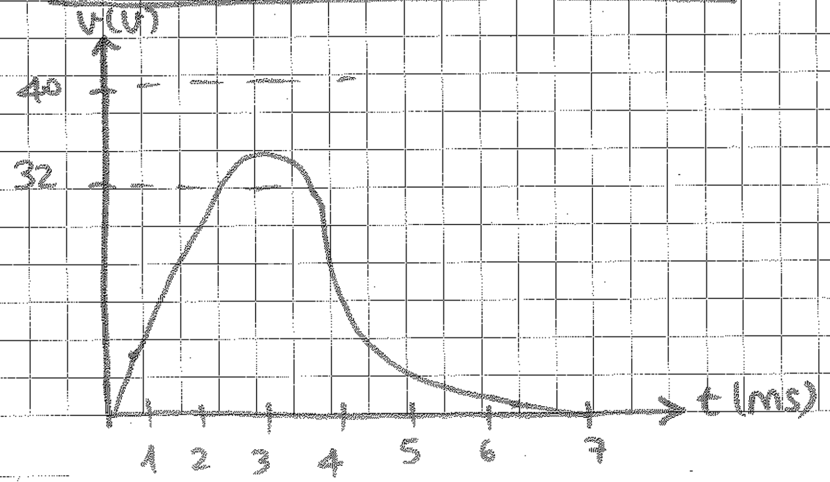
Thus, $\alpha = 10^3 = \frac{1}{2RC} \Rightarrow R = \frac{10^6}{(2000)(0.125)} = \boxed{4k\Omega}$

b) $v(0^+) = 0$

$D_1 = \frac{I_C(0^+)}{C}$ I_L $-(-12.25\,mA)$ Since $i_R(0^+) = 0$ $v_C(0^+) = 0$

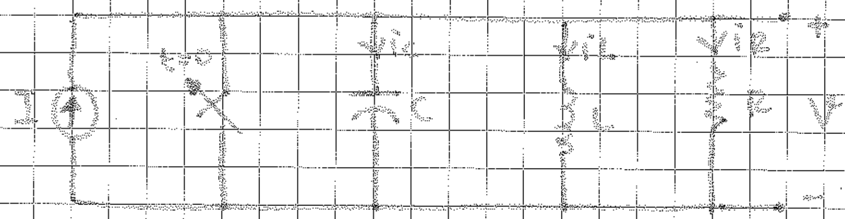
$\frac{dv(0^+)}{dt} = 98.000\ V/s \Rightarrow D_2 = 0,\ D_1 = 98.000\ V/s$

$$\boxed{v(t) = 98000\,t e^{-1000t}\ (V),\quad t \geq 0}$$



STEP RESPONSE OF A PARALLEL RLC-CIRCUIT

→ Consider the following circuit:



→ After applying the KCL and KVL ... (pg. 302)

The governing equation for the circuit is:

$$\boxed{\frac{d^2 i_L}{dt^2} + \frac{1}{RC}\cdot\frac{di_L}{dt} + \frac{i_L}{LC} = \frac{I}{LC}}$$

→ Thus, we need to solve this differential equation.

The solution can be obtained by expressing i_L as a function of v.

→ Then, we have from $v = L\frac{di_L}{dt}$ differentiate both sides,

$$\frac{dv}{dt} = L\cdot\frac{d^2 i_L}{dt^2}$$

→ Thus the circuit equation is:

$$\frac{1}{L}\cdot\frac{dv}{dt} + \frac{1}{RC}\cdot\frac{v}{L} + \frac{1}{L^2 C}\int_0^t v\,d\tau = \frac{I}{LC}$$

→ Multiply both sides by LC

$$\frac{1}{L}\int_0^t v\,d\tau + \frac{v}{R} + C\cdot\frac{dv}{dt} = I$$

→ Differentiate both sides with respect to t

$$\boxed{\frac{v}{L} + \frac{1}{R}\frac{dv}{dt} + C\cdot\frac{d^2 v}{dt^2} = 0} \quad \underline{\underline{OR}} \quad \boxed{\frac{d^2 v}{dt^2} + \frac{1}{RC}\cdot\frac{dv}{dt} + \frac{v}{LC} = 0}$$

whose solution is: $v = A_1 e^{s_1 t} + A_2 e^{s_2 t}$

OR

$v = B_1 e^{-\alpha t}\cos(\omega_d t) + B_2 e^{-\alpha t}\sin(\omega_d t)$

OR

$v = D_1 t e^{-\alpha t} + D_2 e^{-\alpha t}$

depending on the roots s_1 and s_2.

→ The solution for i_L is:

$$i_L = I + A_1' e^{s_1 t} + A_2' e^{s_2 t}$$

OR

$$i_L = I + B_1' e^{-\alpha t}\cos(\omega_d t) + B_2' e^{-\alpha t}\sin(\omega_d t)$$

OR

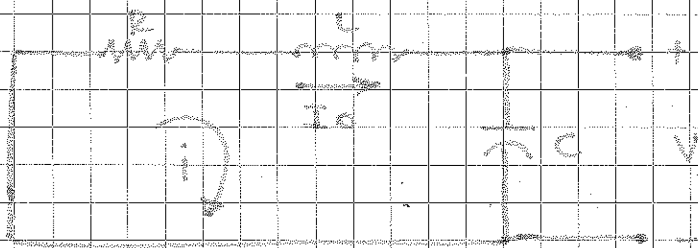
$$i_L = I + D_1' t e^{-\alpha t} + D_2' e^{-\alpha t}$$

Depending on the roots which are s_1 and s_2

(Ex: 8.6) pg 304

THE NATURAL and STEP RESPONSE OF A SERIES RLC CIRCUIT

Consider the circuit:



→ The governing equation is: $Ri + L\dfrac{di}{dt} + \dfrac{1}{C}\displaystyle\int_0^t i\,d\tau + V_0 = 0$

→ Differentiate both sides:

$$R\cdot\frac{di}{dt} + L\frac{d^2 i}{dt^2} + \frac{i}{C} = 0$$

→ Rearranging: $\boxed{\dfrac{d^2 i}{dt^2} + \dfrac{R}{L}\dfrac{di}{dt} + \dfrac{i}{LC} = 0}$

→ The characteristic equation: $\boxed{s^2 + \dfrac{R}{L}s + \dfrac{1}{LC} = 0}$

where, $s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$ where $\alpha = \dfrac{R}{2L}$ and $\omega_0 = \dfrac{1}{\sqrt{LC}}$

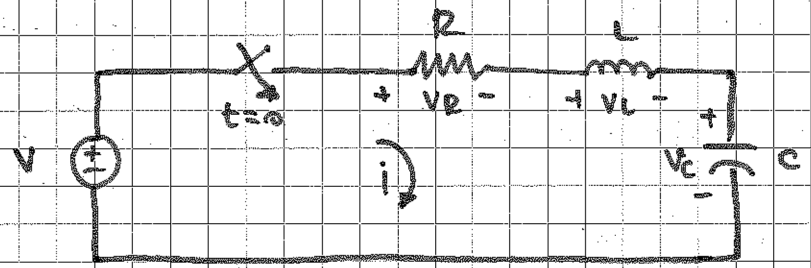
and the solution of the natural response:

$i(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$ (overdamped)

$i(t) = B_1 e^{-\alpha t}\cos(\omega_d t) + B_2 e^{-\alpha t}\sin(\omega_d t)$ (underdamped)

$i(t) = D_1 t e^{-\alpha t} + D_2 e^{-\alpha t}$ (critically damped)

→ In order to find the step response we consider the following circuit



→ After KVL applications we have:

$$\boxed{\frac{d^2 V_C}{dt^2} + \frac{R}{L}\frac{V_C}{dt} + \frac{V_C}{LC} = \frac{V}{LC}}$$

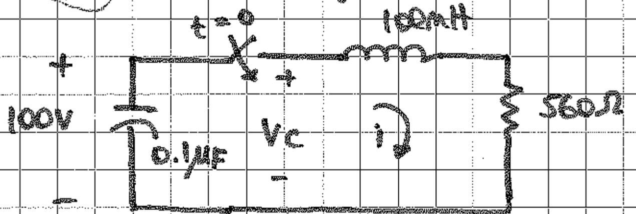
→ The solution is:

$V_C = V_f + A_1' e^{s_1 t} + A_2' e^{s_2 t}$ (overdamped)

$V_C = V_f + B_1' e^{-\alpha t}\cos(W_d t) + B_2' e^{-\alpha t}\sin(W_d t)$ (underdamped)

$V_C = V_f + D_1' t e^{-\alpha t} + D_2' e^{-\alpha t}$ (critically damped)

EX: For the given circuit:



a) Find $i(t)$ for $t \geq 0$

b) Find $V_C(t)$ for $t \geq 0$

Ans a): $W_0 = \frac{1}{LC} = 10^8$ rad/s , $\alpha = \frac{R}{2L} = 2800$ rad/s

Thus, $W_0^2 > \alpha^2 \Rightarrow$ underdamped

$i(t) = B_1 e^{-\alpha t}\cos(W_d t) + B_2 e^{-\alpha t}\sin(W_d t)$ where $\alpha = 2800$ rad/s, $W_d = 98.000$ rad/s

→ B_1 & B_2 are obtained from the initial conditions:

$\boxed{i(0) = 0 = B_1}$

For B_2, we need $\frac{di(0^+)}{dt}$ so, $L\frac{di(0^+)}{dt} = V_0$ or $\frac{di(0^+)}{dt} = \frac{V_0}{L}$

so, $\frac{100}{100} \times 10^3 \longrightarrow \frac{di(0^+)}{dt} = 1000$ A/s

since $B_1 = 0$, $\frac{di}{dt} = 400 B_2 e^{-2800t}(24\cos(9600t) - 7\sin(9600t))$

Thus, $\frac{di(0^+)}{dt} = 9600 B_2 \longrightarrow B_2 = \frac{1000}{9600} \approx 0.1042$ A

Thus, $\boxed{i(t) = 0.1042 e^{-2800t}\sin 9600t, \ t \geq 0}$.

$$\boxed{b)\ V_C(t) = (\underbrace{100}_{\text{since } B_1 = 0}\cos(9600t) + 29.17\sin(9600t) \cdot e^{-2800t} \ (V), \ t \geq 0}$$

09/12/2015

CHAPTER - 9

- SINUSOIDAL STEADY STATE ANALYSIS -

• **The Sinusoidal Source:**

Consider the following voltage source:

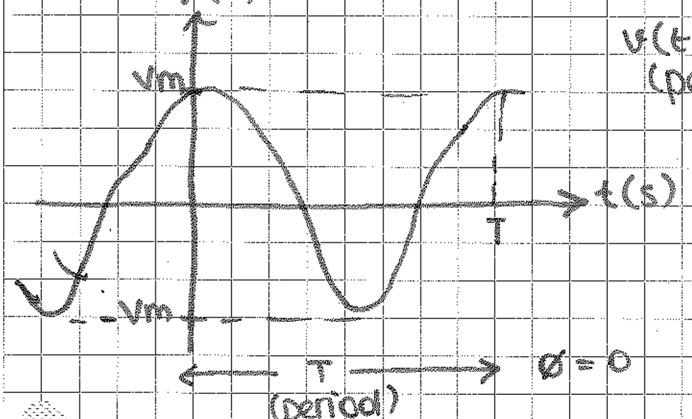
$\boxed{v = V_m \cos(\omega t + \varnothing)}$ where t = time variable (s)
v = voltage (v)
ω = radian frequency (rad/s)
V_m = max. amplitude (V)
$\varnothing$ = phase angle (rad)

sinusoidal voltage source wave form

$\omega = 2\pi f$, f = frequency (Hz),

$T = \frac{1}{f}$ = period (s)

→ If we plot the sinusoidal source wave form w.r.t time

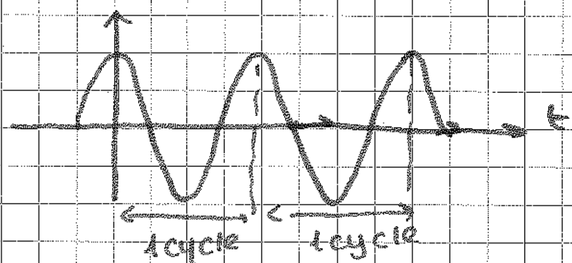


$\varnothing = 0$

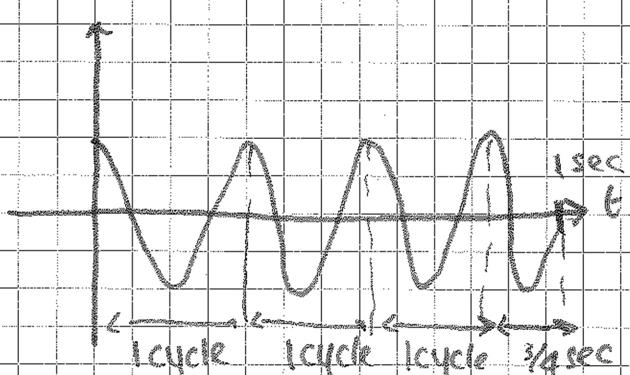
$v(t) = v(t+T)$ (periodic)

Thus,

cycle: the smallest non-repeatable portion of the wave-form

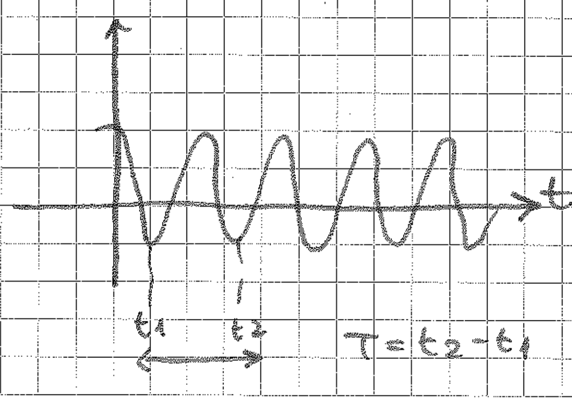


Frequency: Number of cycles in 1 sec



$f = \frac{15}{4}$ Hz

Period = T = time takes for 1 cycle



$T = t_2 - t_1$

• Another important property of sinusoidal voltage (or current) is the "rms value"

(rms : Root Mean Square)

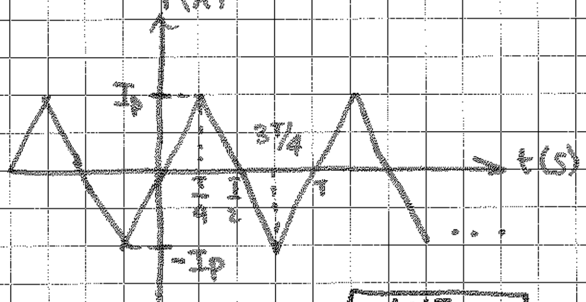
Thus, if $v = V_m \cos(\omega t + \phi)$

$$V_{rms} = \left[\frac{1}{T} \int_{t_0}^{t_0+T} v^2 dt \right]^{1/2} \quad \text{OR} \quad V_{rms} = \sqrt{V_{avg}^2}$$

Mean value of the squared voltage = V_{avg}^2

$$\text{OR} \quad V_{rms} = \frac{1}{T} \int_{t_0}^{t_0+T} V_m^2 \cos^2(\omega t + \phi) dt \longrightarrow \boxed{V_{rms} = \frac{V_m}{\sqrt{2}} \ (V)}$$

× EX: Find the rms value of the periodic triangular function given below:



ANS: $I_{rms} = \sqrt{\frac{1}{T} \int_{t_0}^{t_0+T} i^2 dt}$

Then, $\int_{t_0}^{t_0+T} i^2 dt = 4 \int_0^{T/4} i^2 dt$

From the graph: $i = \underbrace{\frac{4 I_p}{T}}_{\text{slope}} t, \quad 0 < t < T/4$

Substitute the last eqn into the integral,

$$4 \int_0^{T/4} \frac{16 I_p^2}{T^2} t^2 dt = \frac{I_p^2 T}{3}$$

Thus, $i_{mean} = \frac{1}{\cancel{T}} \cdot \frac{I_p^2 \cdot \cancel{T}}{3} = \frac{1}{3} I_p^2$

$$\longrightarrow I_{rms} = \sqrt{\frac{I_p^2}{3}} \longrightarrow \boxed{I_{rms} = \frac{I_p}{\sqrt{3}}}$$