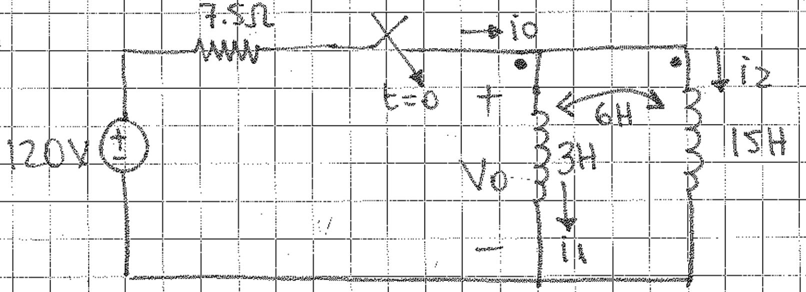
HW

***EX :** For the circuit ;

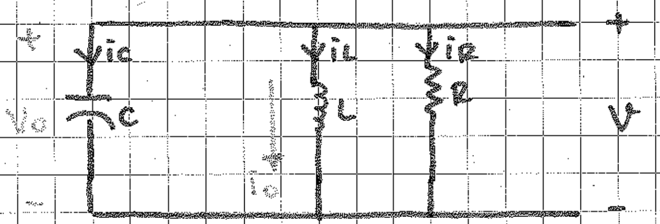


Find i_0, i_1 and i_2 if the initial energy is zero

CHAPTER-8 / RLC-CIRCUITS

NATURAL and STEP RESPONSES of RLC-CIRCUITS

1) Natural Response of Parallel RLC-Circuits :



KCL at the top node gives :

$$\frac{v}{R} + \frac{1}{L}\int_0^t v\,d\tau + I_0 + C\frac{dv}{dt} = 0$$

Differentiate both sides with respect to "t" :

$$\frac{1}{R}\cdot\frac{dv}{dt} + \frac{v}{L} + C\cdot\frac{d^2v}{dt^2} = 0$$

Rearrange the equation :

$$\frac{d^2v}{dt^2} + \frac{1}{RC}\cdot\frac{dv}{dt} + \frac{v}{LC} = 0 \longrightarrow \text{This is a } 2^{nd} \text{ order circuit}$$

The classical approach to solve this diff. eqn is to assume a solution in the form which is exponential:

$v = Ae^{st}$, then substitute the soln into the eqn:

$$As^2e^{st} + \frac{Ase^{st}}{RC} + \frac{Ae^{st}}{LC} = 0 \quad \text{OR} \quad Ae^{st}\left(s^2 + \frac{s}{RC} + \frac{1}{LC}\right) = 0$$

$A=0 \rightarrow$ is a ~~non~~-trivial solution, it cannot be accepted.

Then we have the solution for:

$$\boxed{s^2 + \frac{s}{RC} + \frac{1}{LC} = 0}$$ this is called characteristic eqn

There are 2 roots:

$$s_{1,2} = \frac{-1}{2RC} \pm \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}}$$

Thus,

$v = A_1e^{s_1t}$ and $v = A_2e^{s_2t}$ are the 2 solutions.

Also,

$\boxed{v = A_1e^{s_1t} + A_2e^{s_2t}}$ is a solution.

→ Consider the roots of characteristic eqn is another notation:

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 + w_0^2} \quad \text{where;} \quad \alpha = \frac{1}{2RC} \text{ (Neper frequency)}$$

$$w_0 = \frac{1}{\sqrt{LC}} \text{ (Resonant radian freq.)}$$

*There are 3 possible outcomes:

1) If $w_0^2 < \alpha^2 \rightarrow$ Both roots are real. The response is called overdamped

2) If $w_0^2 > \alpha^2 \rightarrow$ Both roots are complex and conjugate of each others. That is; $s_1 = a_1 + jb_1$ then $s_2 = a_1 - jb_1$ or visa versa. The response is called underdamped

3) If $w_0^2 = \alpha^2 \rightarrow s_{1,2}$ are real and equal. The response is called critically damped

EX: a) For a RLC circuit, with $R = 200\,\Omega$, $L = 50\,mH$, $C = 0.2\,\mu F$. Find the roots of characteristic eqn.

b) Will the response be overdamped, underdamped or critically damped?

c) Repeat (a) and (b) for $R = 312.5\,\Omega$

d) What values of R causes the response to be critically damped?

SOLN: a)

$$\alpha = \frac{1}{2RC} = \frac{10^6}{(400)(0.2)} = 1.25 \times 10^4 \text{ rad/s}$$

$$\omega_0^2 = \frac{1}{LC} = \frac{(10^3)(10^6)}{(50)(0.2)} = 10^8 \text{ rad}^2/s^2$$

then, $s_{1,2} = -1.25 \times 10^4 \mp \sqrt{1.5625 \times 10^8 - 10^8}$

$s_1 = -5000$ rad/s , $s_2 = -20.000$ rad/s

b) Overdamped. Because $\omega_0^2 < \alpha^2$

c) For $R = 312.5\,\Omega \Rightarrow \alpha = 8000$ rad/s , $\alpha^2 = 0.64 \times 10^8$ rad^2/s^2

and $\omega_0^2 = 10^8$ rad^2/s^2

$s_1 = -8000 + j6000$ (rad/s) $s_2 = -8000 - j6000$ (rad/s)
where $j = \sqrt{-1}$

This response is under-damped since $\omega_0^2 > \alpha^2$

d) $\alpha^2 = \omega_0^2 \Rightarrow$ critically damped.

$$\left(\frac{1}{2RC}\right)^2 = \frac{1}{LC} = 10^8 \quad \text{OR} \quad \frac{1}{2RC} = 10^4 \Rightarrow \boxed{R = 250\,\Omega}$$

THE OVERDAMPED VOLTAGE RESPONSE (for the parallel RLC circuit)

The solution is in the form: $v = A_1 e^{s_1 t} + A_2 e^{s_2 t}$ where s_1 and s_2 are the roots of characteristic equation and A_1 and A_2 are constants to be determined by the initial conditions.

Initial conditions: $v(0^+)$ OR $\frac{dv(0^+)}{dt}$

Then, $v(0^+) = A_1 + A_2$ and $\frac{dv(0^+)}{dt} = s_1 A_1 + s_2 A_2$

- The value of $v(0^+)$ is the initial voltage on the capacitor, V_0
- The value of $dv(0^+)/dt$ can be obtained by finding the current in the capacitor. Then,

$$\frac{dv(0^+)}{dt} = \frac{i_C(0^+)}{C}$$

Also;

$$i_C(0^+) = \frac{-V_0}{R} - I_0$$

* SUMMARY OF FINDING OVERDAMPED RESPONSE

1) Find s_1 and s_2
2) Find $v(0^+)$ and $\frac{dv(0^+)}{dt}$
3) Find A_1 and A_2 from $v(0^+) = A_1 + A_2$ and $\frac{dv(0^+)}{dt} = s_1 A_1 + s_2 A_2$
4) Substitute the values into the solution

$$\boxed{v = A_1 e^{s_1 t} + A_2 e^{s_2 t}}$$

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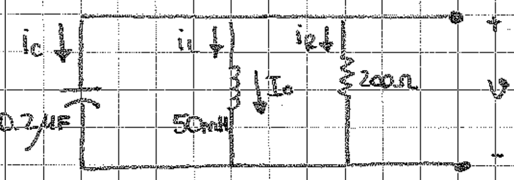
EX: For the circuit below:

$v(0^+) = 12V$ and $I_L(0^+) = 30\,mA$



a) Find the initial current in each branch of the circuit

b) Find the initial value of dv/dt

c) Find the expression for $v(t)$

Ans: a) $i_L(0^-) = i_L(0^+) = i_L(0) = 30\,mA$

$i_R(0^+) = \dfrac{12V}{200\Omega} = \boxed{60\,mA}$

Then, (From the KCL at the top node)

$i_C(0^+) = -i_L(0^+) - i_R(0^+)$

$\boxed{i_C(0^+) = -90\,mA}$ (the direction that we determine for i_C is opposite)

b) $i_C = C \cdot \dfrac{dv}{dt}$ so $\dfrac{dv(0^+)}{dt} = \dfrac{-90\times10^{-3}}{0.2\times10^{-6}} = -450\,\dfrac{kV}{s}$

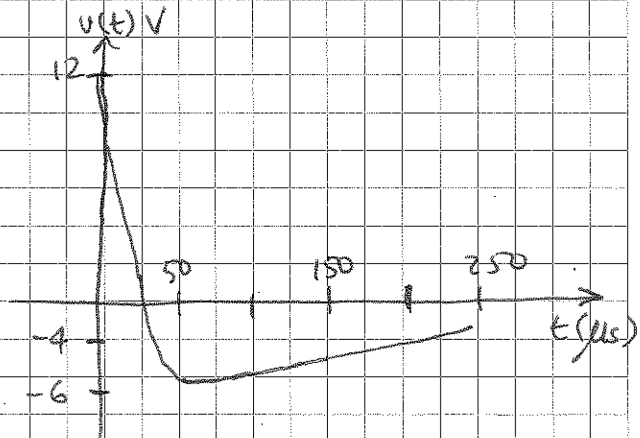
c) Roots of the characteristic equation:

$s_1 = -1.25\times10^4 + \sqrt{1.5625\times10^8 - 10^8}$

$\boxed{s_1 = -5000\ rad/s}$

$s_2 = -1.25\times10^4 - \sqrt{1.5625\times10^8 - 10^8}$

$\boxed{s_2 = -20.000\ rad/s}$



The coefficients are:

$12 = A_1 + A_2$

$-450\times10^3 = -5000A_1 - 20.000A_2$

$\Rightarrow A_1 = -14V$, $A_2 = 26V$

and

$\boxed{v(t) = (-14e^{-5000t} + 26e^{-20000t})\,V,\ t \geq 0}$

THE UNDERDAMPED VOLTAGE RESPONSE

When $\omega_0^2 > \alpha^2$, the roots are:

$$s_1 = -\alpha + \sqrt{-(\omega_0^2 - \alpha^2)} = -\alpha + j\sqrt{\omega_0^2 - \alpha^2}$$

$$\boxed{s_1 = -\alpha + j\omega_d} \quad , \quad \boxed{s_2 = -\alpha - j\omega_d}$$

where;

$\omega_d = \sqrt{\omega_0^2 - \alpha^2}$ is called the damped radian frequency

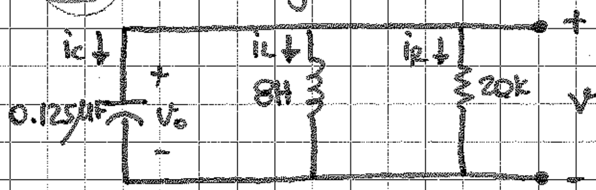
The response is: $\boxed{v(t) = B_1 e^{-\alpha t}\cos(\omega_d t) + B_2 e^{-\alpha t}\sin(\omega_d t)}$

where coefficients B_1 and B_2 can be obtained from:

$$v(0^+) = V_0 = B_1 \quad , \quad \frac{dv(0^+)}{dt} = \frac{i_C(0^+)}{C} = \boxed{-\alpha B_1 + \omega_d B_2}$$

(α: damping factor)

Ex: For the given circuit:



$V_0 = 0V$ and $I_0 = -12.25\,mA$

a) $s_1, s_2 = ?$ b) $v, \frac{dv}{dt}$ at $t = 0^+ = ?$

c) $v(t) = ?$ for $t \geq 0$

Ans: a) $\alpha = \frac{1}{2RC} = 200$ rad/s

$\omega_0 = \frac{1}{\sqrt{LC}} = 10^3$ rad/s

$\frac{-12.25}{125 \times 10^{-7}}$ 10^5

Thus,

$\omega_0^2 > \alpha^2$ (underdamped!)

Now,

$\omega_d = \sqrt{\omega_0^2 - \alpha^2} = 100\sqrt{96} = 979.80$ rad/s

$\boxed{s_1 = -\alpha + j\omega_d = -200 + j\,979.80 \text{ rad/s}}$

$\boxed{s_2 = -\alpha - j\omega_d = -200 - j\,979.80 \text{ rad/s}}$

b) $v(0) = v(0^+) = V_0 = 0$

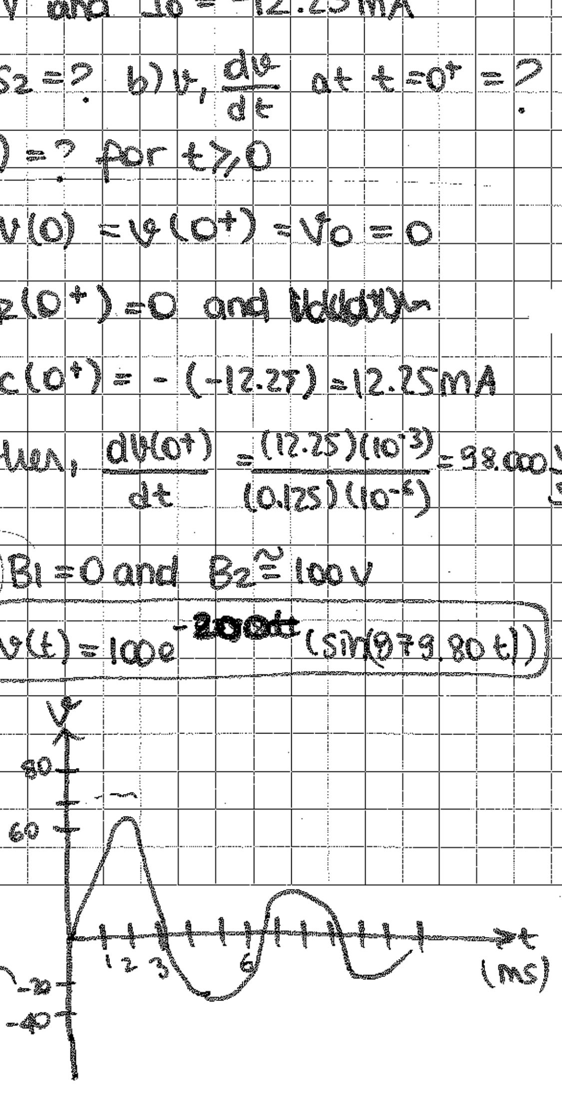
$i_R(0^+) = 0$ and

$i_C(0^+) = -(-12.25) = 12.25\,mA$

then, $\frac{dv(0^+)}{dt} = \frac{(12.25)(10^{-3})}{(0.125)(10^{-6})} = 98.000 \frac{V}{s}$

c) $B_1 = 0$ and $B_2 \cong 100\,V$

$\boxed{v(t) = 100e^{-200t}(\sin(979.80t))}$



Oscillation = Ringing ← Oscillation occurs at freq ω_0